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Browse or Experience

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Abstract. Consumers gain information about the value of a product both prior to purchase and when owning a product. We consider a model where both these types of gaining information are possible. The information gained when owning the product may affect future product purchases. We characterize when the consumer chooses to purchase the product if the consumer does not own it, the expected interval of time between purchases, and the expected number of product purchases over time. We find that keeping product duration fixed, the optimal fixed price is independent of the initial product valuation if that valuation is sufficiently low such that a consumer not owning the product does not purchase it immediately and characterize how the price charged affects the consumer information gathering strategy. We also find that an increase in the information gained while using the product leads to an earlier purchase but less frequent repurchases thereafter. On the other hand, an increase in the information gained prior to purchase leads to later purchases but more frequent repurchases thereafter. When the firm can also choose product duration and there are no costs of production, we find that the firm chooses an expected product duration that is infinitely small and charges a flow price for the consumer to use the product. We also characterize how the extent of learning when owning and when not owning the product, the duration of the product, and the discount rate affect the optimal consumer and firm strategies.

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1. Introduction

Consumers can gain information about the evolving value fit of a product both by checking information about the product prior to owning it and by experiencing the product when owning the product. In the classification of Nelson (1970), for some products all information can be obtained prior to the purchase (search goods), whereas for other products, information is only obtained when using the product (experience goods). In fact, for most products, consumers can gain information on their value both prior to and after purchase.

For example, a consumer receives information about a smartphone by reading about its battery power, display quality, and camera features before owning it but also, receives information after owning it by using the product to take pictures and experiencing its battery life. A video gamer receives information about a gaming console by reading about its processing power, design, and online features before buying it but also, receives information about its ease of use and connectivity after owning it by playing games with friends. This also applies to subscription services. A user receives information about a video streaming service by looking at its catalogue and app features before subscribing to it but also, receives information about the quality of its shows and the accuracy of its recommendation algorithm after becoming a subscriber.

Not all uncertainty can be resolved upon the first purchase. A continuous arrival of information after the initial purchase can affect the consumer's utility from using the product and her subsequent repurchase decisions. For example, a long-time iPhone user may be considering switching to Android after she learns that her favorite game, *Fortnite*, is banned from Apple's app store. A once dismayed PlayStation owner may be interested in the console again after Sony announces a new crossplatform policy that allows her to play with her friends who own other consoles. A Netflix user may stop subscribing after he learns that his favorite show, *The Office*, is no longer available on Netflix but may decide to subscribe again at a later date when he hears his peers praising the new Netflix-exclusive show, *Squid Game*. Today's internet-connected electronics and apps also receive continuous updates on their interface design and features that a consumer may or may not like.

Consider another example. When buying a car, a consumer can gain information on it (including test driving) prior to purchase but will continue to gain information after purchase when using it. Such flow of information both before and after purchase causes a consumer's preferences for the product or service to evolve over time. There is also new information over time because of the changing environment or product features. For example, a consumer's desire to own an SUV (sport utility vehicle) changes as her commute condition or gas price changes. Thus, a consumer's expected utility from owning the product can continue to update even after multiple prior purchases.

We consider a model where consumers can gain information on a product both prior to and after purchase. The product is a durable good that lasts for some uncertain time.[1] If the expected value of the product is sufficiently low and the consumer does not own the product, the consumer may prefer to choose gaining further information about the product until the information is sufficiently positive such that it is worth purchasing the product. However, after purchasing the product, the consumer will continue gaining information about its value, and the consumer will have to make a decision about whether to repurchase when the product breaks down. If the information received when owning the product is predominantly negative, the consumer will hold off on repurchasing the product for some time and will only repurchase it if the consumer receives sufficient positive information while not owning the product to repurchase the product again. If the information received when owning the product is predominantly positive, the consumer will repurchase the product immediately after the product breaks down.

We find that keeping product duration fixed, the optimal constant price for the firm to charge if the expected initial valuation is sufficiently low is independent of the consumer's initial expected valuation. Choosing the price to charge determines the optimal information gathering by consumers, and the firm has to take into account the potential future revenue of the expected future repurchases by the consumer, therefore not lowering the price beyond a certain level, which makes the optimal price not varying with the expected initial valuation if that is sufficiently low.

When allowing for the firm to choose product duration, we find that the firm would prefer an infinitely short product duration under zero marginal costs. Thus, with evolving preferences, the firm prefers renting over selling a durable product, even though there is no issue of price commitment (as in, e.g., Coase 1972, Bulow 1982). By offering a shorter product duration, the firm can better extract the consumer's option value of delaying purchase to obtain information on the product.

In order to obtain these results, we fully characterize the optimal strategy for the consumer of when to purchase the product given its expected value going forward. The model characterizes rich dynamics where a customer may spend some time gaining information before purchasing the product; after a product is purchased and then, breaks down after some time, the consumer can wait a certain period of time to purchase it again or repurchase it immediately. The model also captures the possibility that a consumer continues to derive value from using a product while owning it, even though if the product does break down, the consumer would not immediately repurchase the product. We characterize the certainty equivalent time until the next purchase and the expected number of purchases by a consumer.

We find that a consumer delays her purchase if the product's price is higher, the discount rate is higher, or the expected duration of the product is lower. If the price is greater, the discount rate is greater, or the expected duration of the product is lower, then the present value for the consumer of purchasing the product is lower, and the consumer delays the purchase until the expected valuation of the product is greater.

Potentially more interestingly, the consumer delays purchase when the information gained without owning the product is greater and anticipates the purchase of the product when the information gained while owning/using the product is greater. That is, if more information can be gained by checking the product prior to purchase, the consumer delays purchase until the consumer finds sufficiently good news. If more information can be gained while owning the product, the consumer anticipates the purchase to gain further information on the product. We find that the effect of the greater information obtained prior to purchase dominates the effect of the greater information obtained after purchase, such that when the extent of information gained increases equally prior to and after purchase, the consumer chooses to delay purchase. The information gained prior to purchase allows the consumer to make better decisions immediately in the next purchase occasion. The information gained after the purchase allows the consumer to make better decisions only in future purchase decisions, and therefore, the former effect dominates.

We find that, after the consumer just made an initial purchase, the consumer makes more frequent repurchases when the ratio of the information gained while owning the product to the information gained without owning the product is smaller. That is, an increase in the information gained without owning the product delays the initial purchase but leads to more repurchases after an initial purchase is made. On the other hand, an increase in the information gained while owning the product leads to a faster initial purchase but less frequent repurchases afterward. The consumer is more cautious when buying a search good initially because of the option value of waiting but is quicker to buy it again, whereas the consumer is more eager to buy an experience good initially for its experimentation value but less likely to repurchase it afterward.

This together with the effects on the first purchase can have important implications for the management of the customer lifetime value in a category. In categories in which much of the information is gained when owning the product, the first purchase accounts for a larger share of the customer lifetime value, and managers should focus on generating the first purchase: that is, generating trial. On the other hand, in categories in which much of the information is gained prior to the purchase, the repeat purchases after the initial purchase account for a larger share of the customer lifetime value, and managers should focus on getting consumers to engage in repeated purchases.

When we consider the optimal price charged by the firm, we can then obtain that, in equilibrium, the price and the extent to which the consumer delays the purchase are decreasing in the discount rate and increasing in the expected duration of the product and in the amount of information gained prior to and after purchase. Thus, the effect of price change dominates the effects of changes in the discount rate or product duration. A greater expected duration of the product makes the firm increase the price so much that consumers choose to delay their purchase until they receive sufficient good news. That is, for products of greater duration, consumers wait to receive further information. Also interestingly, when the information gained prior to and after purchase increases, the firm chooses to increase its price, as now, there is a possibility that the consumer gets more positive news about the product.

These findings present important managerial implications for pricing, such that firms should respond with higher pricing to greater consumer patience by consumers or greater product duration, such that consumers are slower to make a purchase. That is, in these changed conditions, it is optimal for firms to delay sales using higher pricing compared with taking advantage of consumers purchasing the product sooner if price stayed fixed or did not increase sufficiently.

When the initial expected valuation by the consumer is high enough, the firm prices the product such that the consumer purchases the product immediately, and in this situation, the optimal price is increasing in the initial expected valuation of the product.

When the firm is also allowed to decide on the product duration under optimal pricing, given the result on the optimal infinitely small product duration, we find that the optimal flow price is increasing in the amount of information gained when not owning the product and decreasing in the discount rate.

The paper presents important managerial implications for firms in settings in which consumers learn information both prior to and after purchase. The paper presents implications both for pricing and for product duration, as described, while at the same time, providing measures of the number of expected sales to a consumer over time.

This paper is related to the existing literature on gaining information prior to choosing an alternative (e.g., Roberts and Weitzman 1981, Moscarini and Smith 2001, Branco et al. 2012, Fudenberg et al. 2018), with the difference that information continues to be gained after the choice is made, which may become useful in future choices.[2] In relation to that literature, the possibility of gaining information after purchase that can become relevant for future purchases makes the consumer anticipate the purchase, and the decision of when to purchase depends on what can be learned after the purchase. The setup presented here allows us to also consider the question of the expected number of purchases by a consumer, whereas the literature looking at information prior to purchase considered only the possibility of either one or zero purchases. Also related to this paper, Chaimanowong and Ke (2022) considers the possibility of the consumer deciding to keep track of product information under occasional repeat purchases. Erdem and Keane (1996) considers empirically a setting in which consumers gain information both when owning and when not owning a product, and consumers make a choice in every period. In relation to that paper, here we concentrate on the effects of owning or not owning a product as well as when to make repurchases, consider optimal pricing and optimal product duration, and obtain general and sharp results on the consumers' and firm's strategic decisions.

There is also literature focusing on information gained only while using the product (e.g., Bergemann and Välimäki 1996).[3] In relation to that literature, this paper considers that the consumer can gain information prior to the purchase, which delays the purchase. There is also the possibility, not considered here, of learning about the quality of products by observing the actions of others and potentially saving on search costs (e.g., Tucker and Zhang 2011, Hendricks et al. 2012).

The remainder of the paper is organized as follows. The next section presents the model, and Section 3 considers the optimal consumer behavior. Section 4 studies the expected length of time between purchases and the expected number of purchases. Section 5 presents the optimal pricing by the firm and the market equilibrium, and it considers the case in which the firm can choose the product duration. Section 6 concludes.

2. The Model

Consider the setting of a risk-neutral consumer who can purchase a product of varying value that lasts for some uncertain period and then, has to decide whether to repurchase it.

Let x be the current expected flow utility generated if the consumer owns the product. With additional information about the product, the current expected flow utility evolves over time. Let x_t be the current expected flow utility of owning the product at time t. Given the definition of additional information gained after time t, we know that x_t is a martingale, $E(x_{t+\Delta}|x_t) = x_t$, for any $\Delta > 0$. This also implies that the increments in x_t over time are uncorrelated. (Table 1 presents the notation used throughout the paper.)

Consider the following simple example to illustrate the forces at work. Suppose that the consumer knows that the product is worth at least three but is uncertain about an attribute that could be worth an additional either zero or two with equal probability. At time 0, before knowing the value of the attribute, the expected flow utility of the product for the consumer, x_0, is $4 = 3 + \frac{1}{2}0 + \frac{1}{2}2$. After learning the value of the product, the expected flow utility at time 1, x_1, is either three or five with equal probability. This illustrates how x is a martingale, $E(x_1|x_0) = x_0$. Consider now the extreme case in which the consumer only learns the value of the attribute if the consumer owns the product. Then, if the consumer does not own the product at time 0, the expected flow utility will continue to be $x_0 = 4$ at time 1. On the other hand, if the consumer owns the product at time 0, the expected flow utility at time 1 will be either $x_1 = 5$ or $x_1 = 3$.[4] That is, if the consumer gains more information about

Table 1. Notation

Variable	Description
x	Current expected flow utility of using the product
$\overline{x}$	Purchase threshold for the expected flow utility
σ^2	Extent of learning information while not owning the product
s^2	Extent of learning information while owning the product
λ	Hazard rate at which the product breaks down
P	Price of purchasing the product
r	Continuous discount rate used by consumers and the firm
$\widetilde{r}$	Actual discount rate
β	Hazard rate at which a consumer exits the market
$W(x)$	Expected present value of payoffs for the consumer when not owning the product
$V(x)$	Expected present value of payoffs for the consumer when owning the product and $x \geq \overline{x}$
$\widetilde{V}(x)$	Expected present value of payoffs for the consumer when owning the product and $x < \overline{x}$
μ	$\sqrt{2r/\sigma^2}$
$\widetilde{\mu}$	$\sqrt{2r/s^2}$
$\widehat{\mu}$	$\sqrt{2(r+\lambda)/s^2}$
$\delta(x)$	Expected discount factor of the time until the next purchase when not owning the product
$T(x)$	Certainty equivalent time until the next purchase when not owning the product
$\widetilde{\delta}(x)$	Expected discount factor of the time until the next purchase when owning the product
$\widetilde{T}(x)$	Certainty equivalent time until the next purchase when owning the product
$N(x)$	Expected number of purchases going forward when not owning the product
$\widetilde{N}(x)$	Expected number of purchases going forward when owning the product
η	$\sqrt{2\beta/\sigma^2}$
$\widetilde{\eta}$	$\sqrt{2\beta/s^2}$
$\widehat{\eta}$	$\sqrt{2(\beta+\lambda)/s^2}$
$G(x)$	Expected discounted number of units purchased going forward when not owning the product
$\widetilde{G}(x)$	Expected discounted number of units purchased going forward when owning the product
$\overline{X}$	$e^{\mu\overline{x}}$
$\widetilde{X}$	$e^{\widetilde{\mu}\overline{x}}$
$\widehat{X}$	$e^{\widehat{\mu}\overline{x}}$
x^*	Solution to Equation (27)
$h(\overline{x}, x_0)$	Notation representing Equation (31)
x^{**}	Solution to $h(\overline{x}, \overline{x}) = 0$
$\Pi(x)$	Expected present value of profits
A	$\frac{\widetilde{\mu}}{\widehat{\mu}}\frac{\mu-\widehat{\mu}}{\mu+\widetilde{\mu}}\frac{(r+\lambda)(s^2/\sigma^2-1)}{\lambda-r(s^2/\sigma^2-1)}$

the product when owning it than when not owning it, then the variance of the expected flow utility over time is greater when owning the product than when not owning it.[5] We study the case where the consumer does not receive all information about product utility immediately upon using it, so the consumer's expected flow utility from using the product continues to evolve as the consumer receives more information while owning the product.

We consider that the additional information arrives continuously over time to the consumers. This can also be seen as the limit when information arrives in discrete time periods and the length of the time period goes to zero. This allows us to obtain sharper results. In particular, we assume that x evolves continuously over time as a Brownian motion with zero drift, with potentially varying variance depending on whether the consumer currently owns the product. The flow utility of not owning the product is set at zero.[6]

The consumer could be an owner of the product, in which case she can potentially gain some information over time about the product value. Also, the consumer may not own the product but may be learning the value of the product. We interpret the evolution of x as learning on a number of attributes for a certain period of time. Alternatively, we can also interpret the evolution of x as learning about changes in preferences if preferences evolve over time. We discuss possible specifics of what consumers learn in the online appendix and show cases where the information gained has constant variances when owning and not owning the product. Note that the presentation in terms of attributes is only to illustrate how the information can be gained over time. In fact, the setup considered here is consistent in any way in which information is gained over time under the constraint that the information gained per unit of time is constant (to make the analysis tractable).

Let σ^2 be the variance of the Brownian motion when the consumer does not own the product but is learning information about the product, and let s^2 be the variance of the Brownian motion when the consumer owns the product. We may expect that the consumer learns more attributes when owning the product than when not owning the product, which means that $s^2 \geq \sigma^2$. We consider the general case of $s^2 \neq \sigma^2$, but one interesting benchmark is the case of $s^2 = \sigma^2$, such that the preferences evolve in the same way whether the consumer owns or does not own the product.

When the consumer owns the product, we allow for the possibility of the consumer not using the product. That is, when the current utility x is negative, the consumer chooses not to use the product, as using it is detrimental with respect to not using it and getting an instant utility of zero. We assume that the consumer, while owning the product, learns at the same rate s^2 whether using or not using the product. One may consider that the extent of learning when owning but not using the product is smaller than the extent of learning when using the product. However, note that the extent of learning while owning but not using the product may be significantly greater than when not owning the product. That is, the extent of learning while owning but not using the product may be closer to the extent of learning while using the product than to the extent of learning when not owning the product. In order to simplify the analysis and not to add an additional parameter on the extent of learning while owning but not using the product, we set that extent of learning to be the same as when using the product, s^2. Considering the extent of learning while owning but not using the product as an additional parameter can be done in a relatively straightforward way in the analysis that follows.

Let λ be the hazard rate at which the product breaks down, and let P be the price of purchasing the product. When the product breaks down, the consumer can decide to not repurchase the product immediately and learn about the product with informativeness σ^2, or the consumer can repurchase the product immediately and continue to learn about the product with informativeness s^2.

Consumers and the firm discount the future at the continuous discount rate r. Note that at state x, the expected value of owning the product going forward is $\frac{x}{r+\lambda}$. In Section 4, we consider the possibility that r is composed by an actual discount rate $\widetilde{r}$ and a hazard rate β of the consumer exiting the market, with $r = \widetilde{r} + \beta$. This hazard rate of the consumer exiting the market would be seen as the consumer not needing the product anymore in his lifetime. For example, a consumer at some point may no longer have a use for an automobile and does not have a need to purchase any additional automobile, or the consumer at some point may no longer need a gaming console. This hazard rate of the consumer exiting the market could also be seen as the consumer passing away. This distinction between $\widetilde{r}$ and β is only relevant in Section 4 when we study the expected number of purchases in the lifetime of a consumer in this market. In the remainder of the paper, the only relevant construct is $\widetilde{r} + \beta$, which we define as r.

The optimal strategy of the consumer is going to be characterized by an $\overline{x}$ such that if the consumer does not own the product, the consumer chooses to buy the product if $x \geq \overline{x}$.

We can see the discount rate r as playing the role of the costs of learning information (information-processing costs). The discount rate makes the consumer willing to purchase immediately if the product provides a sufficiently high value, as delaying the purchase just delays the benefits of owning the product. Note, however, that a greater discount rate also makes the present value of owning the product lower, which may make the consumer more demanding on the value at which to purchase the product.

2.1. Further Discussion of Assumptions

Regarding the assumptions on learning prior to and after purchase, we could also think that some random attributes that were discovered in the past stop mattering as time goes by. In many cases, the information gained during learning can have a decreasing variance over time, which can be seen as an intermediate situation between the case considered here and a situation where the consumer learns everything at the first encounter with the product.[7] In this way, the case considered here can be seen as the extreme case in which the importance of information gained is constant. Alternatively, we could think of a situation of evolving preferences, where preferences may evolve at different speed when the consumer owns/uses the product than when the consumer does not own the product. Note also that the setup presented allows for learning both prior to and after purchase and does not depend on the attributes that can be learned through product experience after purchase being the same as the attributes that can be learned prior to purchase. That is, the setup is consistent, with consumers being able to learn some attributes before purchase and only being able to learn some other attributes after purchase.

The role of the information gained with either experience or browsing only affects consumer decision making in the model through the changes to the expected flow utility. It may be that the information gained with experience or browsing is actually on different pieces of information. For example, some attributes can only be learned when using the product. This distinction can potentially have significant effects in different setups, but in the model considered here with an infinite number of attributes (or of available information), the effect of this distinction can only be captured by the different variance of the information gained while browsing or using the product.

We consider that the consumer does not have any costs of obtaining information and that information comes to the consumer freely, even when not owning the product. This can be seen as the situation of the consumer getting information from friends or from media without acting to get that information. This assumption allows us to fully characterize the optimal strategy of the consumer by a unique threshold $\bar{x}$ for the expected current utility x at which the consumer decides to purchase the product if the consumer does not own the product yet, which simplifies the analysis. Were the consumer to also have costs and the decision of whether to keep learning information, the optimal strategy of the consumer would then need to be characterized with an additional threshold, below which the consumer would decide not to gather information on the product (see Branco et al. 2012). This additional threshold, such that consumers would leave the market when this threshold is reached, could provide a justification for consumers to leave the market (as the hazard rate of the consumers exiting the market β mentioned), but it creates substantial complications in the analysis.

3. Optimal Consumer Behavior

Consider now the optimal consumer behavior of when to purchase or delay purchase. Let $W(x)$ be the expected present value of payoffs for the consumer if the consumer does not own the product and is getting information on the product, $x < \bar{x}$. Let $V(x)$ be the expected present value of payoffs for the consumer if the consumer owns the product and $x \geq \bar{x}$. Let $\widetilde{V}(x)$ be the expected value of payoffs for the consumer if the consumer owns the product and $x < \bar{x}$. We focus the initial presentation on the case in which at the optimum $\bar{x} > 0$, which will occur if $s^2 = \sigma^2$ or the product's price is high enough. We later consider also the situation in which the optimum has $\bar{x} < 0$.

When the consumer does not own the product and is searching for information, we can obtain that the evolution of $W(x)$ is characterized by

$$W(x) = e^{-r\,dt} E[W(x + dx)]. \tag{1}$$

Using Itô's lemma, we can get $rW(x) = W''(x)\frac{\sigma^2}{2}$. Note also that $\lim_{x\to-\infty} W(x) = 0$, as the present value of benefits has to approach zero when the current utility derived of potentially using the product goes to negative infinity. We can then obtain the solution for $W(x)$, presented in (A.10) in the online appendix.

When the consumer owns the product and $x \geq \bar{x}$, we have that the expected present value of consumer payoffs has to satisfy

$$\begin{aligned} V(x) = x\,dt + e^{-r\,dt}\lambda\,dt\{E[V(x + dx)] - P\} \\ + e^{-r\,dt}(1 - \lambda\,dt)E[V(x + dx)], \end{aligned} \tag{2}$$

where the first term represents the expected flow utility of owning the product. The second term represents the possibility of the product breaking down, which occurs with probability $\lambda\,dt$, in which case the consumer buys the product again immediately with an expected net benefit of $E[V(x + dx)] - P$, and the third term represents the possibility of the product not breaking down, in which case the consumer gets the expected present value of consumer payoffs if owning the product after the evolution in x, $E[V(x+dx)]$. Using Itô's lemma, (2) reduces to $rV(x) = x - \lambda P + V''(x)\frac{s^2}{2}$. Note that $\lim_{x\to\infty}[V(x) - (x-\lambda P)/r] = 0$, as when the current utility goes to infinity, the consumer is always buying the product when it breaks down, which generates an expected utility of $(x-\lambda P)/r$. Using this when solving the differential equation on $V(x)$, one obtains $V(x)$ as a function of one constant to be determined, presented in (A.11) in the online appendix.

Consider now that the consumer owns the product and $x < \bar{x}$. We consider that the consumer can choose not to use the product if $x < 0$, so we will further divide this region into $(0, \bar{x})$ and $(-\infty, 0]$. Consider first the case of $x \in (0, \bar{x})$. In this region, we have

$$\widetilde{V}(x) = x\,dt + e^{-r\,dt}\lambda\,dt W(x) + e^{-r\,dt}(1-\lambda\,dt)E\widetilde{V}(x+dx), \tag{3}$$

where the first term represents the expected flow utility of owning the product. The second term represents the possibility of the product breaking down, which occurs with probability $\lambda\,dt$, in which case the consumer gets the expected present value of consumer payoffs if not owning the product, $W(x)$, and the third term represents the possibility of the product not breaking down, in which case the consumer gets the expected present value of consumer payoffs if owning the product after the evolution in x, $E[\widetilde{V}(x+dx)]$. Using Itô's lemma and solving the resulting differential equation, one obtains the solution for $\widetilde{V}(x)$, presented in (A.12) in the online appendix.

For the case of $x \le 0$, we can similarly obtain

$$\widetilde{V}(x) = e^{-r\,dt}\lambda\,dt W(x) + e^{-r\,dt}(1-\lambda\,dt)E\widetilde{V}(x+dx). \tag{4}$$

Note also that $\lim_{x\to-\infty}\widetilde{V}(x) = 0$, as the expected utility when owning the product goes to zero when the current utility of using the product approaches negative infinity. Using this, when solving for (4), we can obtain $\widetilde{V}(x)$, presented in (A.13) in the online appendix.

Value matching and smooth pasting at both $\bar{x}$ and zero, $W(\bar{x}) = V(\bar{x}) - P, W'(\bar{x}) = V'(\bar{x}), V(\bar{x}) = \widetilde{V}(\bar{x}), V'(\bar{x}) = \widetilde{V}'(\bar{x}), \widetilde{V}(0^+) = \widetilde{V}(0^-)$, and $\widetilde{V}'(0^+) = \widetilde{V}'(0^-)$ allow us then to determine the constants of integration and fully obtain $W(x), V(x)$, and $\widetilde{V}(x)$. Value matching and smooth pasting at $\bar{x}$ and zero guarantee that $\bar{x}$ is the optimal threshold for the consumer to choose to purchase the product if the consumer does not own the product (e.g., Dixit 1993).

We can then obtain (derivation presented in the online appendix)

$$\begin{aligned}&\widehat{\mu}\bar{x} + e^{-\widehat{\mu}\bar{x}} - 1 - \widehat{\mu}P(r+\lambda)\\ &= (\mu - \widehat{\mu})(r+\lambda)\frac{r(s^2/\sigma^2 - 1)}{\lambda - r(s^2/\sigma^2-1)}\\ &\left[\frac{\widetilde{\mu}}{\mu+\widetilde{\mu}}\frac{\bar{x}-\lambda P}{r} - \frac{\widetilde{\mu}}{\mu+\widetilde{\mu}}P + \frac{1}{r(\mu+\widetilde{\mu})}\right],\end{aligned} \tag{5}$$

which determines $\bar{x}$, where $\mu = \sqrt{2r/\sigma^2}, \widetilde{\mu} = \sqrt{2r/s^2}$, and $\widehat{\mu} = \sqrt{2(r+\lambda)/s^2}$.

Note that if $s^2 = \sigma^2$, (5) reduces to

$$\widehat{\mu}\bar{x} + e^{-\widehat{\mu}\bar{x}} - 1 - \widehat{\mu}P(r+\lambda) = 0, \tag{6}$$

from which we can obtain that the threshold $\bar{x}$ increases in P, r, λ, and σ^2, under the constraint $s^2 = \sigma^2$. Given the focus on learning information about the product both prior to and after the purchase, this case can be seen as reasonable to consider. We also discuss the case in which s^2 is much greater than σ^2, which in the limit is the case of experience goods, that is, the case with just learning information when owning the product.[8] Comparing (6) with (5), we can see that if $\sigma^2 < s^2$ but $(s^2 - \sigma^2)$ is small, we can have that $\bar{x}$ is decreasing in s^2. It is also interesting to observe that when $s^2 = \sigma^2$, the consumer does not purchase immediately when the present value of the current utility is equal to the price. That is, $\frac{\bar{x}}{\lambda+r} > P$. This is because the consumer wants to keep the option of not purchasing the product alive a little longer. The consumer wants to see if the expected current utility is sufficiently large before deciding to make the purchase.

Note that when $s^2 > \sigma^2$ and the price is relatively low, we may have $\bar{x} < 0$. The consumer would buy even though the consumer does not intend to use it immediately because the consumer can get more information about the product by owning it. The optimal behavior in that case is derived in the online appendix. The consumer buys when $\bar{x}$ reaches

$$e^{\mu\bar{x}} = P\left[\widetilde{\mu}(\lambda+r) + r\widetilde{\mu}\frac{\mu+\widetilde{\mu}}{\widetilde{\mu}-\mu}\frac{r(1-s^2/\sigma^2)}{r(1-s^2/\sigma^2)+\lambda}\right]. \tag{7}$$

Note that if s^2 and σ^2 are close, (7) can be approximated by

$$e^{\mu\bar{x}} = P\widetilde{\mu}(\lambda+r), \tag{8}$$

from which we can obtain that the threshold $\bar{x}$ increases in P, r, λ, and σ^2 and decreases in s^2, as in the case of $x > \bar{x}$. We state these results in the following proposition.

Proposition 1. *Suppose that the information gained prior to and the information gained after purchase are close to each other, s^2 close to σ^2. Then, the threshold to purchase* *the product, $\bar{x}$, increases in the price charged, P, in the discount rate, r, in the hazard rate of the product breaking down, λ, in the amount of information gained without owning the product, σ^2, both when s^2 stays fixed and when $s^2 = \sigma^2$, and decreases in the amount of information gained while owning the product, s^2.*

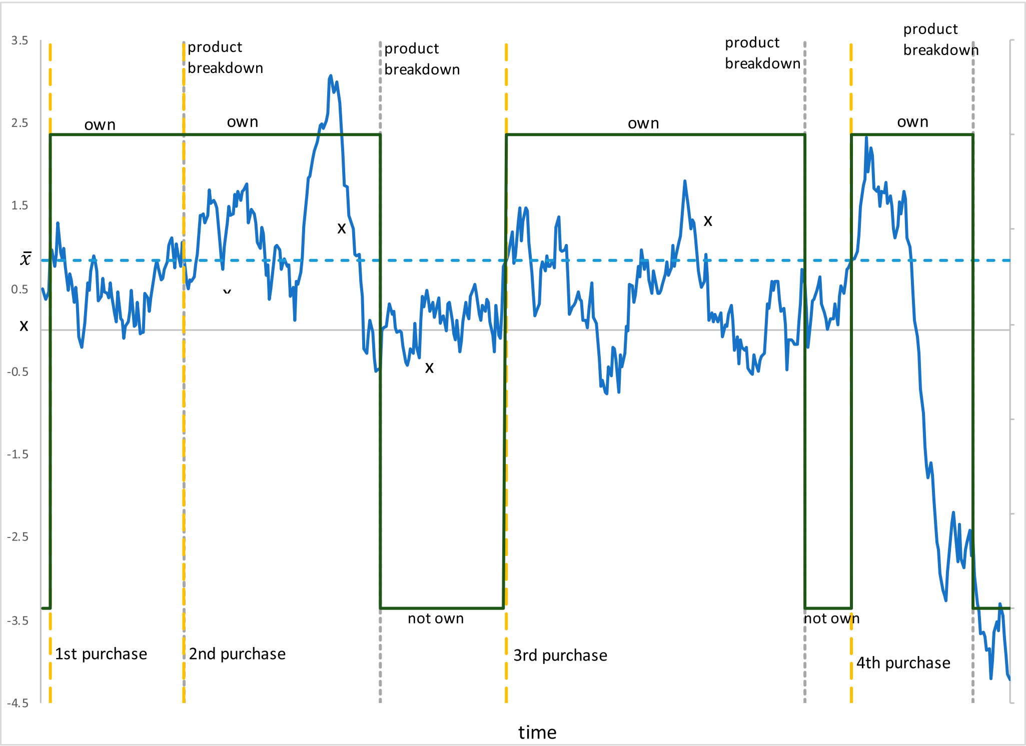
When the present value of payoffs to the consumer of acquiring the product declines, which can occur when there is an increase in either the price or the discount rate or a decrease in the expected duration of the product (greater λ), the consumer is more demanding on the expected current utility of using the product before deciding to purchase it (a greater $\bar{x}$). The effect of the discount rate on the purchase threshold is interesting. On the one hand, a greater discount rate makes delaying having the benefits of owning the product more costly, which is a force toward decreasing the purchase threshold. On the other hand, as noted, a greater discount rate lowers the present value of the benefits of owning the product, which is a force to increase the purchase threshold. The proposition shows that the latter effect dominates the former.

When the amount of information gained without owning the product, σ^2, increases, the consumer chooses to gain further information about the high current utility that the product can deliver, and $\bar{x}$ increases. When the amount information gained when owning the product, s^2, increases, the consumer chooses to anticipate the purchase of the product in order to gain further information for future repurchase, and $\bar{x}$ decreases. The former effect dominates, as it relates to the immediate purchase; when both σ^2 and s^2 increase in the same amount, we have that the consumer delays purchase, and $\bar{x}$ increases. If we interpret the model as a consumer with evolving preferences, then the consumer delays the purchase when her preferences are more volatile.

An increase in s^2 can also be seen as approaching the case in which the consumer learns substantially after purchase, and therefore, the case with lower s^2 may lead to a higher purchase threshold than the case in which the consumer learns everything immediately after purchase. If there is substantial learning after the purchase, the consumer is more likely to anticipate the purchase to learn more.

As an example of the value of $\bar{x}$ as it relates to the different parameters, for $P = 2, r = 0.05, \lambda = 0.2, \sigma^2 = 0.2$, and $s^2 = 0.25$, we can obtain $\bar{x} \approx 0.84$. Figure 1 illustrates a sample path with repeated purchases and product breakdowns given the evolution of preferences for these parameter values. Numerical analysis also suggests that the comparative statistics presented

Figure 1. (Color online) Example of the Sample Path of Consumer Expected Current Utility with Repeated Purchases and Product Breakdowns for $p = 2, r = 0.05, \lambda = 0.02, \sigma^2 = 0.2$ and $s^2 = 0.25$



Note. For these parameter values, we have $\bar{x} \approx 0.84$.

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in Proposition 1 for the case when s^2 is close to σ^2 also hold for general s^2 and σ^2.

3.1. Extreme Case of Experience Goods

It is also interesting to consider the case in which the information gained when owning the product, s^2, is much greater than the information gained when not owning the product, σ^2. The extreme case is the one in which $\sigma^2 \to 0$, when we are in the experience goods case. In this case, if $x < \bar{x}$ and the consumer does not own the product, the consumer never purchases the product. If $x \geq \bar{x}$ and the consumer does not own the product or the product breaks down, the consumer buys the product immediately. Note that after owning the product, if x is below $\bar{x}$ when the product breaks down, the consumer does not purchase the product ever again.

For the case in which the price P is not too low, we can obtain from (5) that the purchase threshold in the limit is determined by

$$\left(\widehat{\mu} + \widetilde{\mu}\frac{r+\lambda}{r}\right)[\bar{x} - P(r+\lambda)] + e^{-\widehat{\mu}\bar{x}} + \frac{\lambda}{r} = 0. \qquad (9)$$

One can then obtain that $\bar{x}$ increases in P, decreases in s^2, and increases in λ and r when r is sufficiently large, which is as in Proposition 1. Numerical analysis suggests that these comparative statics also hold for r small.

For the case in which the price P is sufficiently low, we can obtain from (7) that $\bar{x} \to 0$, and furthermore,

$$\lim_{\sigma^2 \to 0} \frac{\bar{x}}{\sigma} = \frac{1}{\sqrt{2r}} \ln\{P[\widetilde{\mu}(r+\lambda) - \widehat{\mu} r]\}. \qquad (10)$$

Again, one can confirm that for σ^2 small, $\bar{x}$ increases in P, λ, and r and decreases in s^2, as in Proposition 1.

3.2. Extreme Case of Search Goods

Consider now the case in which the consumer gains information prior to purchase and after owning the product, the consumer will buy it again immediately once it breaks down. This case can be seen as the case of search goods and can be obtained with $\sigma^2 > 0$ and $s^2 = 0$. In such a case, when x reaches $\bar{x}$, the consumer purchases the product, and the product value stops updating. When the product breaks down, the consumer buys again immediately, as $x = \bar{x}$. Thus, when the consumer purchases the product, the consumer receives a total discounted utility of $[x - (r+\lambda)P]/r$, where x/r is the present value of utility from using the product and $(r+\lambda)P/r$ is the present value of current and future prices that the consumer pays.

Value matching and smooth pasting at $\bar{x}$, $W(\bar{x}) = [x - (r+\lambda)P]/r$, and $W'(\bar{x}) = 1/r$, where $W(x)$ is given in (A.10) in the online appendix, allow us to determine $\bar{x}$. We get

$$\bar{x} = \sqrt{\frac{\sigma^2}{2r}} + (r+\lambda)P, \qquad (11)$$

from which it is straightforward to check that $\bar{x}$ increases in P, σ^2, r, and λ, as in Proposition 1.

4. Expected Time to Next Purchase and Expected Number of Purchases

4.1. Expected Time to Next Purchase

In this section, we consider some properties in terms of the timing of purchases and the expected number of purchases given the optimal consumer behavior. Consider the certainty equivalent time until the next purchase associated with the expected discount factor of the length of time until the next purchase.[9] That is, if δ is the expected discount factor of the time until the next purchase, then $T = -\frac{1}{r}\ln\delta$ is the certainty equivalent time until the next purchase.

Consider first the case in which the consumer does now own the product. In that case, if $x \geq \bar{x}$, the consumer purchases the product immediately, and the discount factor of the length of time until the next purchase is one (the certainty equivalent time until the next purchase is zero). Consider now that $x < \bar{x}$. Let $\delta(x)$ be the expected discount factor of the time until the next purchase, and let $T(x)$ be the certainty equivalent time until the next purchase.

We have that

$$\delta(x) = e^{-r\,dt} E\delta(x + dx), \qquad (12)$$

from which, using Itô's lemma, value matching at $\bar{x}, (\delta(\bar{x}) = 1)$, and that the expected discount factor for $x \to -\infty$ approaches zero, $(\lim_{x\to-\infty}\delta(x) = 0)$, one can obtain

$$\delta(x) = e^{-\mu(\bar{x} - x)}, \qquad (13)$$

which yields $T(x) = \mu(\bar{x} - x)/r$. We recall that $\mu = \sqrt{2r/\sigma^2}$.

Consider now the case when the consumer owns the product. Let $\widetilde{\delta}(x)$ be the expected discount factor of the time until the next purchase, and let $\widetilde{T}(x)$ be the certainty equivalent time until the next purchase.

If $x > \bar{x}$, we have that the evolution of $\widetilde{\delta}(x)$ over time has to satisfy

$$\widetilde{\delta}(x) = \lambda\,dt + (1 - \lambda\,dt)e^{-r\,dt} E\widetilde{\delta}(x + dx). \qquad (14)$$

Note also that $\lim_{x\to\infty}\widetilde{\delta}(x) = \frac{\lambda}{\lambda+r}$, as when the current utility approaches infinity, the product will be almost surely repurchased when it breaks down, and the expected discount factor of duration of the product given that it is functioning is $\frac{\lambda}{\lambda+r}$.

Consider now the case of $x < \bar{x}$. The evolution of $\widetilde{\delta}(x)$ over time has to satisfy

$$\widetilde{\delta}(x) = \lambda\, dt\, \delta(x) + (1 - \lambda\, dt)e^{-rdt} E\widetilde{\delta}(x + dx). \tag{15}$$

Note that as $x \to -\infty$, the expected discount factor until the next purchase approaches zero. Using the value of $\widetilde{\delta}(x)$ as x converges to plus and minus infinity, Itô's lemma on (14) and (15), and value matching and smooth pasting at $\bar{x}$, $\widetilde{\delta}(\bar{x}^+) = \widetilde{\delta}(\bar{x}^-)$, and $\widetilde{\delta}'(\bar{x}^+) = \widetilde{\delta}'(\bar{x}^-)$, we can then obtain $\widetilde{\delta}(x)$, and therefore, we can obtain $\widetilde{T}(x)$ (see the online appendix for the derivation).

When the consumer just purchased the product, we can compute the certainty equivalent time until the next purchase as

$$\widetilde{T}(\bar{x}) = -\frac{1}{r}\ln\left[\frac{\lambda}{2(\lambda + r)} + \frac{\lambda(1 - \mu/\widehat{\mu})}{2[\lambda + r(1 - s^2/\sigma^2)]}\right]. \tag{16}$$

We can then obtain the following proposition characterizing the certainty equivalent time between two purchases.

Proposition 2. *After the consumer just made a purchase, the certainty equivalent time to the next purchase is increasing in the expected duration of the product,* $1/\lambda$, *and in the amount of information gained while owning the product,* s^2, *and is decreasing in the amount of information gained when not owning the product,* σ^2, *and in the discount rate,* r.

As the amount of learning when the consumer owns the product, s^2, is larger, the current utility can have greater negative shocks, leading the consumer not to repurchase the product immediately after the products breaks down. Similarly, as the amount of learning when the consumer does not own the product, σ^2, is larger, the current utility evolves faster while not owning the product, which could potentially lead to the consumer repurchasing the product sooner.

Interestingly, note that the effect of s^2 on the certainty equivalent time to repurchase is different than its effect on the certainty equivalent time to the initial purchase. From Proposition 1, we see that a larger s^2 decreases $\bar{x}$, which shortens the certainty equivalent time if $x < \bar{x}$. Thus, when there is more information when owning the product, the consumer is more eager to make the initial purchase, but after making the initial purchase, such a consumer is slower to make subsequent purchases. This also means that the composition of customer lifetime value depends on the extent to which the product is an experience good. An increase in s^2 increases the initial purchase's contribution to the customer lifetime value while decreasing the proportion of the customer lifetime value derived from repeated purchases.

The effect of the expected duration of the product is also interesting. A greater expected duration of the product makes the certainty equivalent time to the next purchase increase, as the product lasts longer. However, the effect of product duration on the certainty equivalent time to repurchase is different than its effect on the certainty equivalent time to the initial purchase. From Proposition 1, a greater expected duration of the product makes the purchase threshold lower, which would be a force toward decreasing the certainty equivalent time to purchase if the consumer does not own the product. A greater discount rate decreases the certainty equivalent time to the next purchase as the discount factor for any time horizon decreases in the discount rate.

For the example considered of $\lambda = 0.2, \sigma^2 = 0.2$, $s^2 = 0.25$, and $r = 0.05$, we have that the certainty equivalent time until the next purchase after one purchase is 8.1 units of time compared with the expected duration of the product of 5 units of time. Figure 2 illustrates the evolution of the certainty equivalent time until the next purchase when not owning, $T(x)$, and when owning the product, $\widetilde{T}(x)$, as a function of the current utility of having the product, x.

As one would expect, the certainty equivalent time until the next purchase is decreasing in the current expected utility. As the expected current utility of the product is lower, the consumer is more likely to delay the purchase of the product once it breaks down.

4.2. Expected Number of Future Purchases

To construct the expected number of purchases given the optimal consumer behavior, we have to use the hazard rate β of the consumer dropping out of the market. Recall that the discount rate r considered was composed of both the actual discount rate $\widetilde{r}$ and the hazard rate of the consumer dropping out of the market, $r = \widetilde{r} + \beta$. This hazard rate β of the consumer dropping out of the market captures the idea that consumers end up making a finite number of purchases in a category over their lifetime. In the remainder of the paper, this hazard rate can be folded into an overall discount rate r, but to study the actual number of purchases done by a consumer, it has to be explicitly considered. We expect β to be much smaller that λ, the hazard rate of the product breaking down, such that a consumer makes multiple purchases over the consumer's lifetime.

Let $N(x)$ be the expected number of units purchased going forward given that the consumer starts at $x < \bar{x}$ and the consumer does not own the product. We have that $N(x)$ evolves over time as

$$N(x) = (1 - \beta\, dt)EN(x + dx). \tag{17}$$

Let $\widetilde{N}(x)$ be the expected number of future units purchased over time given that the consumer owns the

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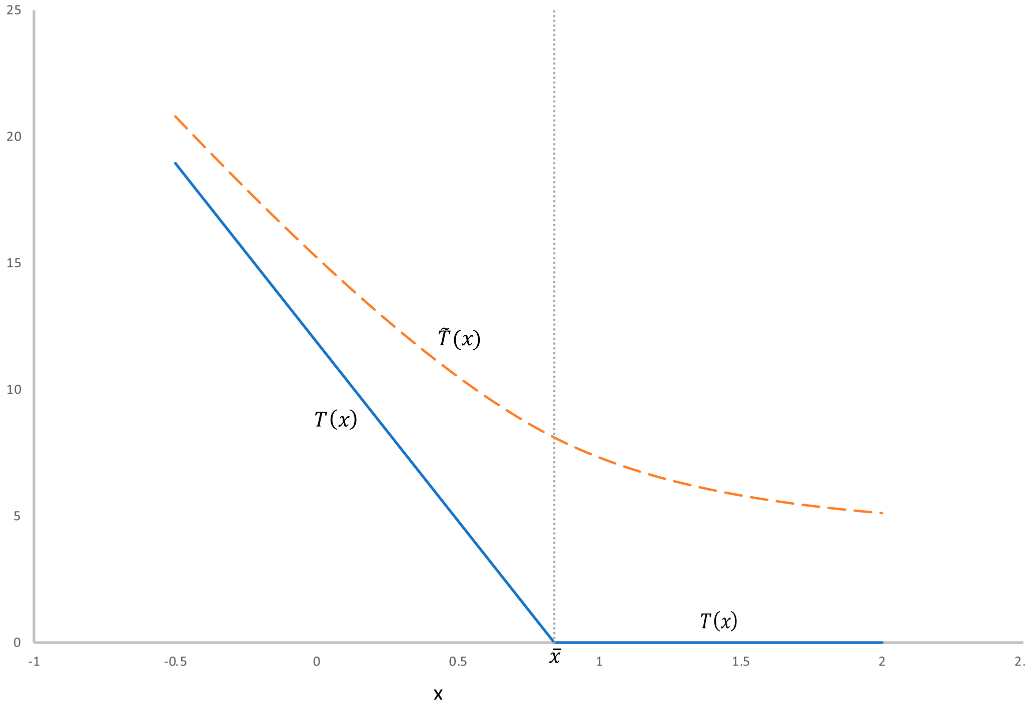
Figure 2. (Color online) Evolution of the Certainty Equivalent Time Until the Next Purchase When Not Owning, $T(x)$, and When Owning the Product, $\widetilde{T}(x)$, as a Function of the Current Utility of Having the Product, x, for $p = 2$, $r = 0.05, \sigma^2 = 0.2$, $s^2 = 0.25$, and $\lambda = 0.2$



product. As the consumer purchases the product immediately if the consumer does not own the product and $x = \bar{x}$, we have

$$N(\bar{x}) = 1 + \widetilde{N}(\bar{x}). \quad (18)$$

For $x \geq \bar{x}$, the evolution of $\widetilde{N}(x)$ over time has to satisfy

$$\widetilde{N}(x) = \lambda\, dt\,[1 + \widetilde{N}(x)] + (1 - \lambda\, dt - \beta\, dt) E\widetilde{N}(x + dx). \quad (19)$$

Consider now the evolution of $\widetilde{N}(x)$ for $x < \bar{x}$. This yields

$$\widetilde{N}(x) = \lambda\, dt\, N(x) + (1 - \lambda\, dt - \beta\, dt) E\widetilde{N}(x + dx). \quad (20)$$

Applying Itô's lemma on (17), (19), and (20), solving the corresponding differential equations, and using value matching and smooth pasting at $\bar{x}$ for $\widetilde{N}(x)$, $\widetilde{N}(\bar{x}^+) = \widetilde{N}(\bar{x}^-)$, and $\widetilde{N}'(\bar{x}^+) = \widetilde{N}'(\bar{x}^-)$, we can then obtain the expected number of purchases going forward when $x < \bar{x}$ as (see the online appendix for the derivation)

$$N(x) = e^{\eta(x-\bar{x})}\left[1 + \frac{\widehat{\eta}s^2/\sigma^2 - \eta(\lambda+\beta)/\beta}{\widetilde{\eta}\frac{\beta(1-s^2/\sigma^2)}{\lambda} + \widetilde{\eta}\left(\frac{\lambda+\beta(1-s^2/\sigma^2)}{\lambda}\right) + \eta} + \frac{\lambda}{\beta}\right], \quad (21)$$

where $\eta = \sqrt{2\beta/\sigma^2}$, $\widetilde{\eta} = \sqrt{2\beta/s^2}$, and $\widehat{\eta} = \sqrt{2(\beta+\lambda)/s^2}$.

When $s^2 = \sigma^2$, this expression simplifies to

$$N(x) = \frac{1}{2} e^{\eta(x-\bar{x})}\left[\frac{\beta+\lambda}{\beta} + \sqrt{\frac{\beta+\lambda}{\beta}}\right]. \quad (22)$$

We can then obtain the following result.

Proposition 3. *Suppose that the amount of information gained without owning or while owning the product is not too different, s^2 close to σ^2. Then, the expected number of purchases going forward after the consumer just made a purchase is decreasing in the expected duration of the product, $1/\lambda$, in the hazard rate of the consumer dropping out of the market, β, and in the ratio s^2/σ^2. Starting from an initial current utility $x < \bar{x}$, the expected number of purchases going forward is decreasing in the price charged, P, in the actual discount rate, $\widetilde{r}$, and in the hazard rate of the consumer dropping out of the market, β, and is increasing (decreasing) in the amount of information gained under the constraint $s^2 = \sigma^2$ if the initial current utility is low (high) enough. The expected number of purchases at $x < \bar{x}$ is increasing in the expected duration of the product, $1/\lambda$, if the hazard rate of the consumer dropping out of the market is not too low.*

As one would regard as likely, the expected number of purchases going forward is decreasing in the hazard rate of the consumer dropping out of the market and in the price charged. More interestingly, an increase in the actual discount rate $\widetilde{r}$ leads to a lower expected number of purchases as the consumer discounts more the future benefits and has, therefore, a lower present value of the benefits of buying the product. This then makes the consumer more demanding on the current expected utility of the product to decide to purchase the product. This results in a lower expected number of purchases going forward.

The effect of the amount of information gained before and after purchase on the expected number of purchases depends on the initial expected current utility because of two conflicting market forces. On the one hand, a greater amount of information gained allows the current utility to move substantially over time, and this is a force for the expected number of purchases to increase when the initial current utility is very low. On the other hand, a greater amount of information gained makes the consumer more demanding in terms of current utility in order to decide to purchase the product (greater $\bar{x}$), and this is a force to reduce the expected number of purchases. The former market force dominates if the initial current utility is very low, whereas the latter dominates if the initial current utility is not too low.

The effect of the ratio s^2/σ^2 on the expected number of repurchases going forward after a purchase is negative. An increase in the ratio s^2/σ^2 means that the consumer after each purchase has a likelihood of receiving negative information through using the product, which may be difficult to recover from when the product breaks down, yielding a lower number of purchases going forward. This means that there are more repeated purchases when the product is more of a search good than when the product is more of an experience good after the consumer makes an initial purchase. A manager should expect more sales to come from returning consumers if the product is more of a search good and expect more sales to come from new consumers if the product is more of an experience good. This insight has implications on the relative importance of customer acquisition versus retention.

The effect of the expected duration of the product also has opposing market forces on the expected number of purchases. On the one hand, after a purchase, a longer expected duration of the product leads to a lower number of purchases going forward. On the other hand, initially, a longer duration of the product makes the consumer more willing to purchase the product and less demanding on the current utility (lower $\bar{x}$), which will lead to more purchases. Which market force dominates depends on the hazard rate of the consumer dropping out of the market, with an

Figure 3. (Color online) Evolution of the Expected Number of Purchases Going Forward When Not Owning ($N(x)$) and When Owning the Product ($\widetilde{N}(x)$) as a Function of the Current Utility of Having the Product, x, for $p = 2$, $r = 0.05, \beta = 0.02, \sigma^2 = 0.2$, $s^2 = 0.25$, and $\lambda = 0.2$

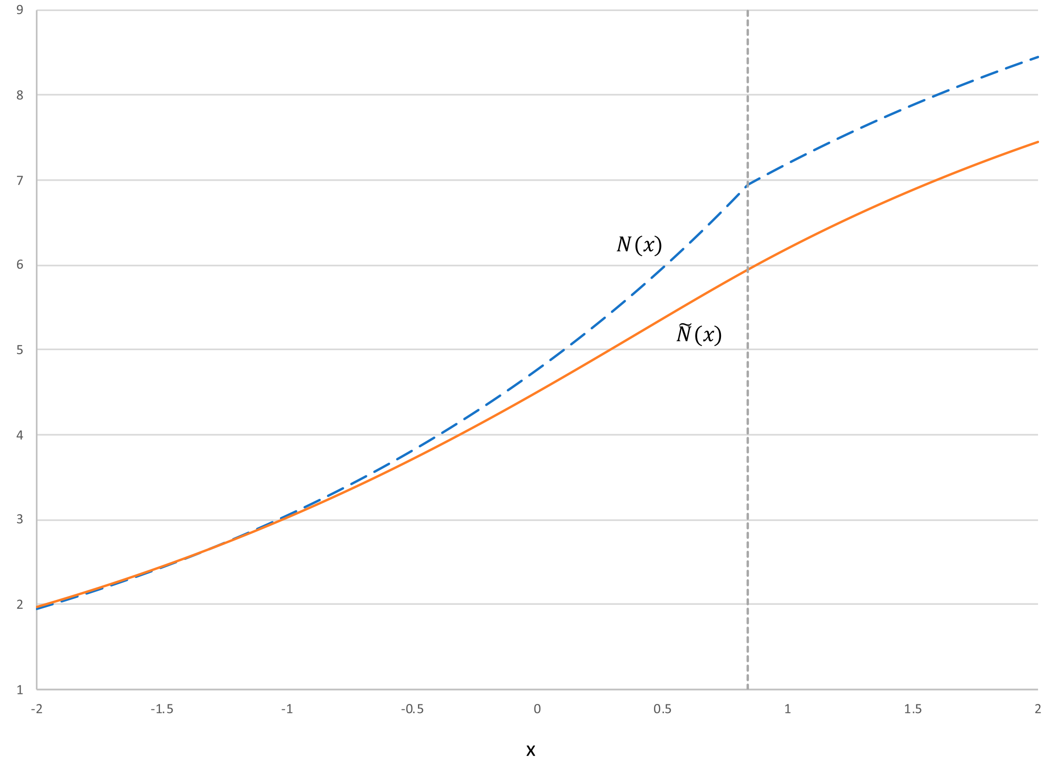
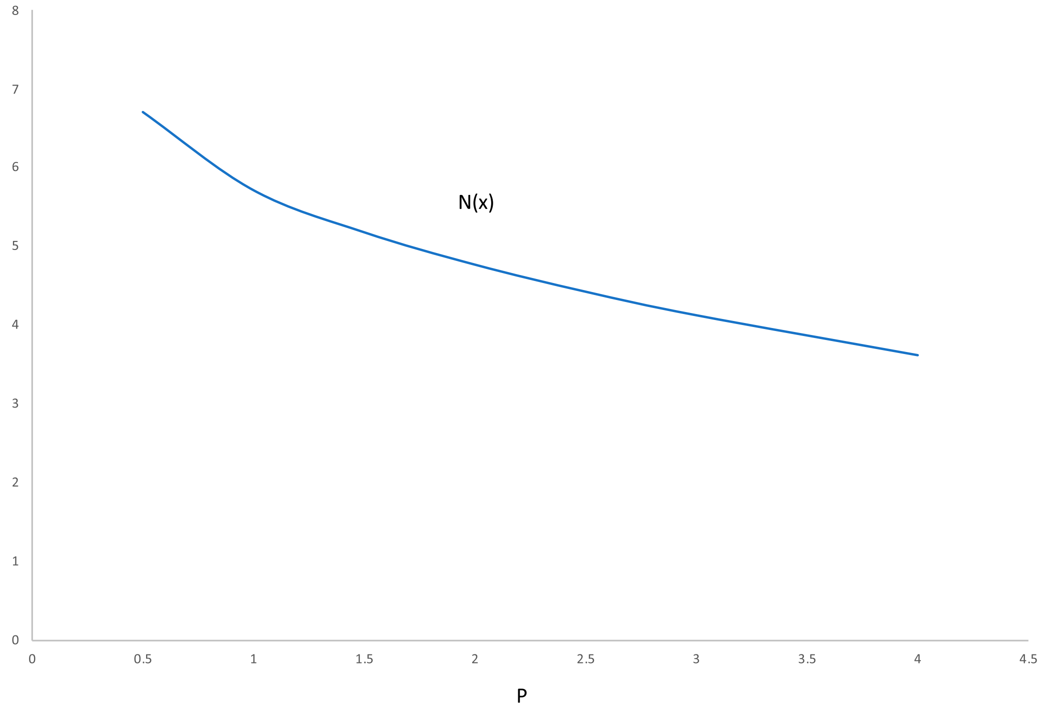


Figure 4. (Color online) Evolution of the Expected Number of Purchases Going Forward When Not Owning the Product, $N(x)$, as a Function of the Price Charged P for $x = 0$, $\widetilde{r} = 0.03, \beta = 0.02, \sigma^2 = 0.2, s^2 = 0.25$, and $\lambda = 0.2$



increase in the expected duration of the product leading to more purchases if the hazard rate of the consumer dropping out of the market is not too low.

Figure 3 illustrates how the expected numbers of purchases going forward when not owning, $N(x)$, and when owning the product, $\widetilde{N}(x)$, evolve as a function of the current utility x. Note that $N(x) = \widetilde{N}(x) + 1$ for $x > \bar{x}$, as in that case, the consumer purchases the product immediately. Note that for $x < \bar{x}$, we have $N(x) < \widetilde{N}(x) + 1$, as in that region of x, there is uncertainty that the consumer will make another purchase. Figures 4–9 illustrate the results in the proposition on how the expected number of purchases going forward evolves with the price charged, P, the actual discount rate, $\widetilde{r}$, the hazard rate of the consumer dropping out of the market, β, the amount of information gained without owning the product, σ^2, the amount of information gained when owning the product, s^2, and the hazard rate of the product breaking down, λ, respectively.

In particular, Figure 7 illustrates that depending on the value of the current utility x, the effect of the amount of information gained when not owning the product on the expected number of purchases going forward can be positive or negative. As discussed, for a current utility that is not too low (the case of $x = 0$ in Figure 7), a greater amount of information gained makes the consumer more demanding on the expected valuation needed to trigger a purchase, which decreases the expected number of purchases. This effect is reduced by the effect that immediately after purchase, the expected number of purchases going forward decreases in the ratio s^2/σ^2. For a current utility that is relatively low (the case $x = -2$ in Figure 7), a greater amount of information gained makes the consumer more likely to reach the purchase threshold, and in that case, the expected number of purchases going forward increases.

Figure 8 illustrates that an increase in the amount of information gained when owning the product increases the expected number of purchases going forward, but as argued for the purchase threshold, this effect is smaller than the effect of the information gained when not owning the product; therefore, we obtain the result in the proposition that when the amounts of information while owning or not owning the product are the same and there is an increase in the same amount on both types of information, the effect of σ^2 dominates, and we have the pattern of Figure 7.

Figure 9 illustrates how the effect of the expected duration of the product on the expected number of purchases going forward can be positive or negative, with it being positive if the hazard rate of the consumer dropping out of the market is not too low.

Figure 5. (Color online) Evolution of the Expected Number of Purchases Going Forward When Not Owning the Product, $N(x)$, as a Function of the Actual Discount Rate $\widetilde{r}$ for $x = 0,\ p = 2, \beta = 0.02, \sigma^2 = 0.2, s^2 = 0.25$, and $\lambda = 0.2$

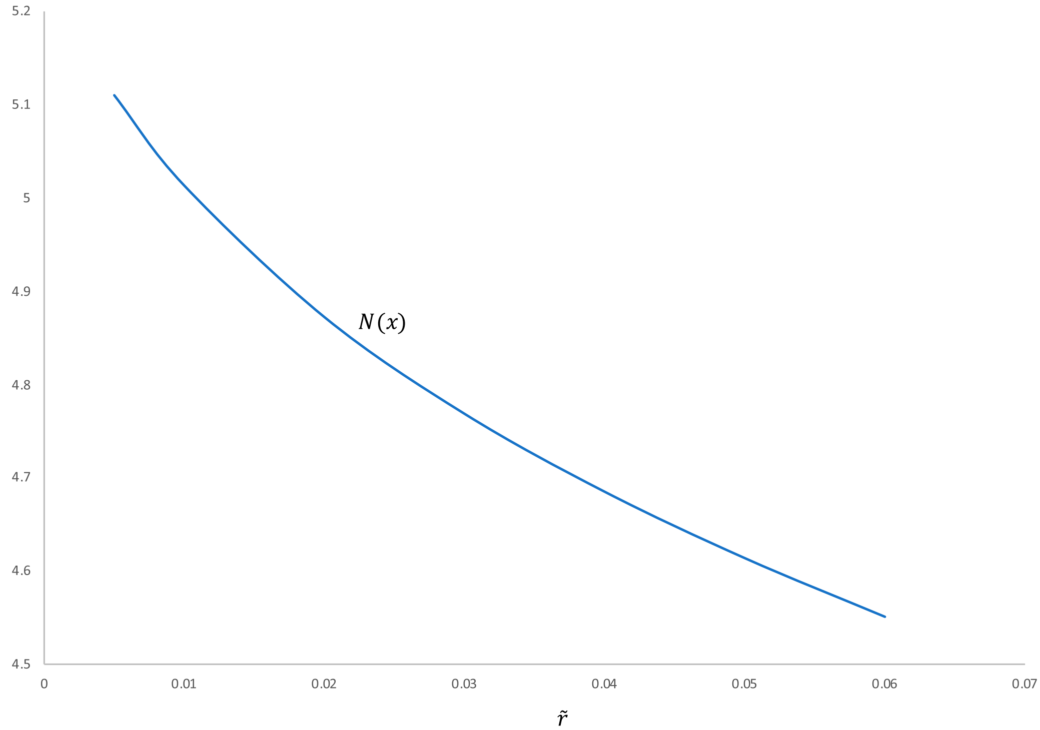


Figure 6. (Color online) Evolution of the Expected Number of Purchases Going Forward When Not Owning the Product, $N(x)$, as a Function of β for $x = 0,\ p = 2, \widetilde{r} = 0.03, \sigma^2 = 0.2, s^2 = 0.25$, and $\lambda = 0.2$

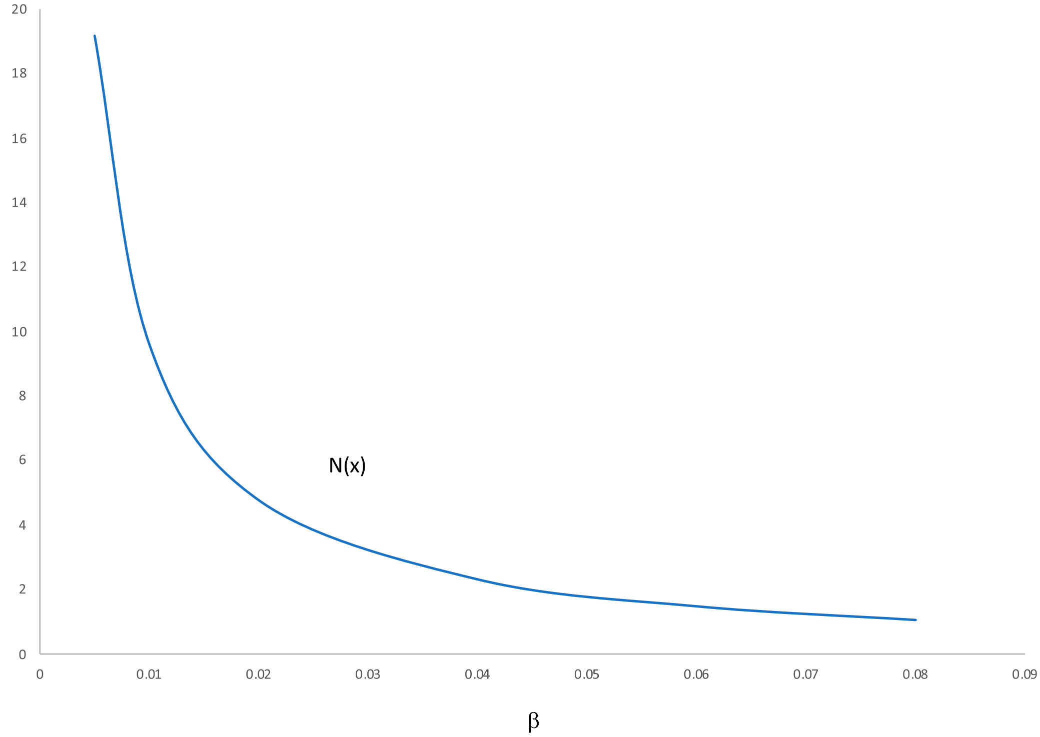


Figure 7. (Color online) Evolution of the Expected Number of Purchases Going Forward When Not Owning the Product, $N(x)$, as a Function of σ^2 for $x = 0$ and $x = -2$, with $p = 2, \widetilde{r} = 0.03, \beta = 0.02, s^2 = 0.25$, and $\lambda = 0.2$

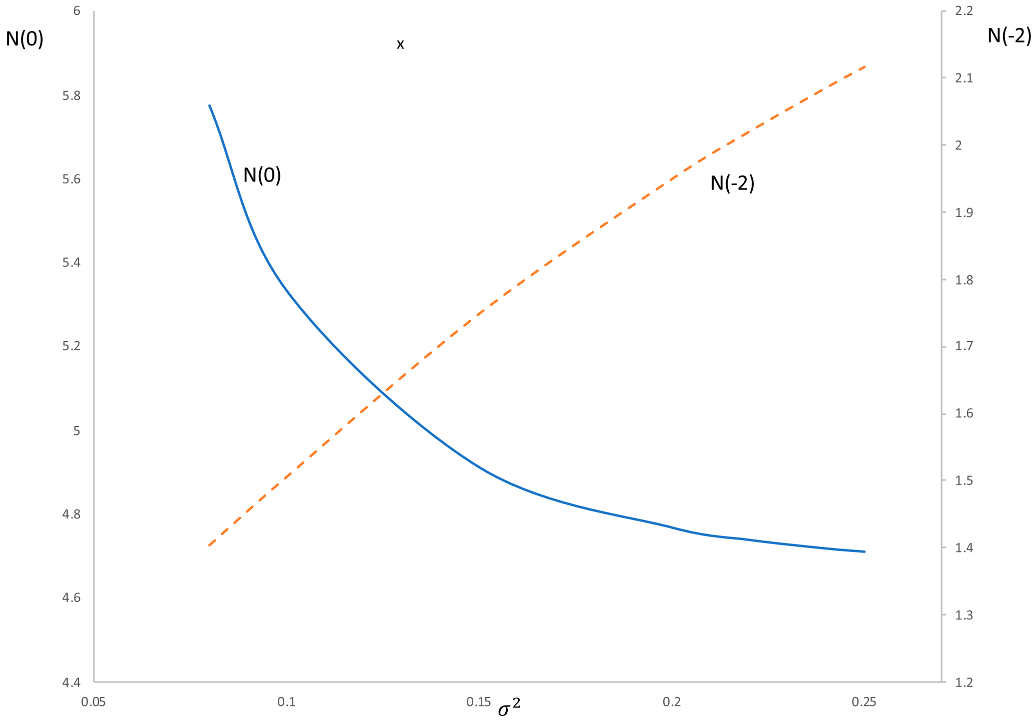


Figure 8. (Color online) Evolution of the Expected Number of Purchases Going Forward When Not Owning the Product, $N(x)$, as a Function of s^2 for $x = 0$ and $x = -2$, with $p = 2, \widetilde{r} = 0.03, \beta = 0.02, \sigma^2 = 0.2$, and $\lambda = 0.2$

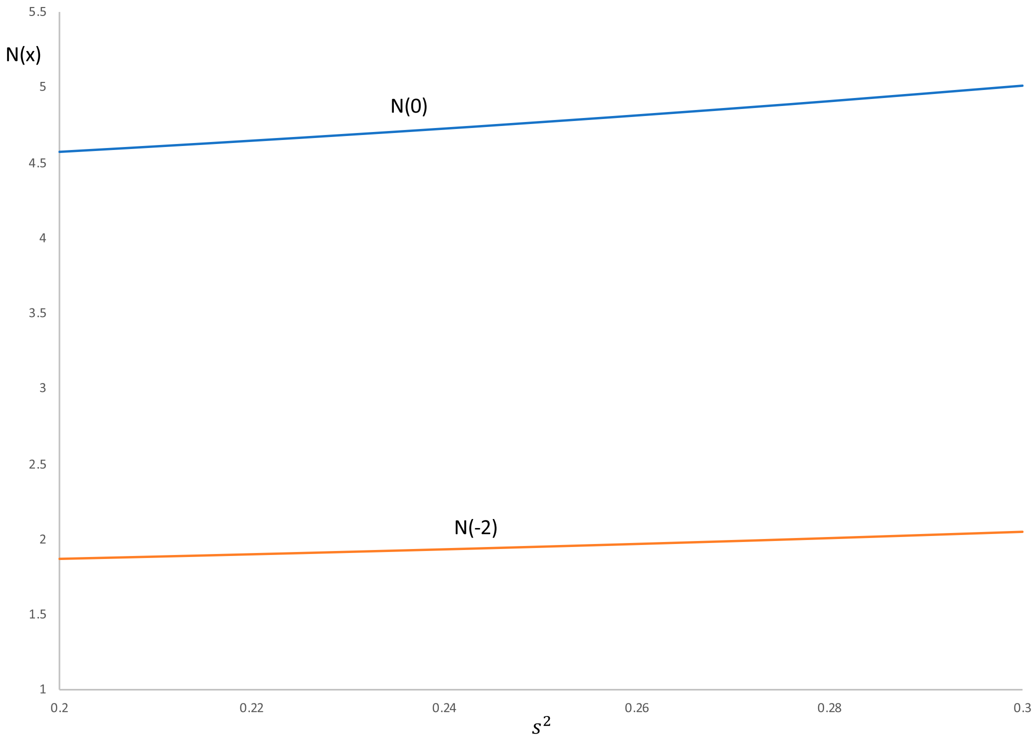
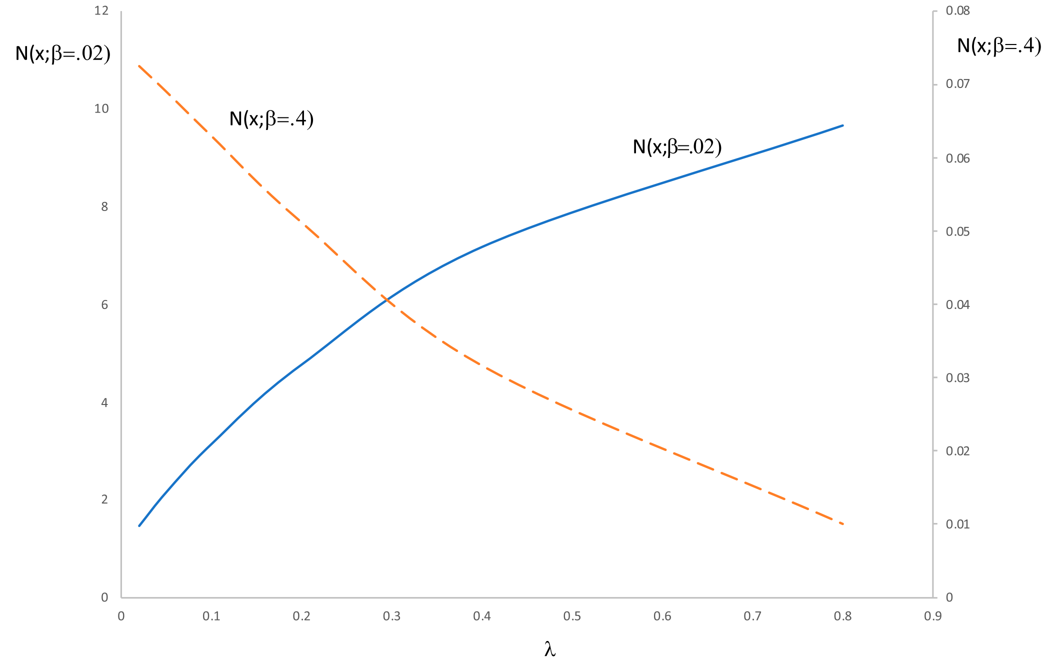


Figure 9. (Color online) Evolution of the Expected Number of Purchases Going Forward When Not Owning the Product, $N(x)$, as a Function of λ for $\beta = 0.02$ and $\beta = 0.4$, with $x = 0$, $p = 2, \widetilde{r} = 0.03, s^2 = 0.25$, and $\sigma^2 = 0.2$



5. Optimal Pricing and Market Equilibrium

5.1. Optimal Pricing

Consider now the optimal pricing of a firm, assuming that the price chosen is fixed over time and that the optimal price is chosen to maximize the present value of expected profits over time given that the consumers start with an expected current utility x_0. To simplify the presentation, we consider that when the firm makes the pricing decision, all consumers have the same starting expected current utility x_0. Alternatively, the firm can have a prior distribution over x_0 and do optimal pricing given that prior distribution over x_0, with effects that are similar to the ones considered here.[10]

To construct the present value of profits, let $G(x)$ be the discounted number of units purchased over time given that the consumer starts at $x < \overline{x}$ and the consumer does not own the product and $\widetilde{G}(x)$ be the discounted number of units purchased over time given that the consumer owns the product. The construction of $G(x)$ and $\widetilde{G}(x)$ is similar to the construction of $N(x)$ and $\widetilde{N}(x)$ in Section 4.2, and it is presented in the online appendix.[11] In particular, we can obtain

$$G(x) = e^{\mu(x-\overline{x})}\left[1 + \frac{\widehat{\mu}s^2/\sigma^2 - \mu(\lambda+r)/r}{\widehat{\mu}\dfrac{r(1-s^2/\sigma^2)}{\lambda} + \widetilde{\mu}\left(\dfrac{\lambda + r(1-s^2/\sigma^2)}{\lambda}\right) + \mu} + \frac{\lambda}{r}\right]. \tag{23}$$

Suppose that the consumer does not buy at $t = 0$. That is, $x_0 < \overline{x}$. As optimizing on the price to charge, under zero production costs, is $\max_P PG(x_0)$, we can obtain the first-order condition as

$$1 - \mu P\frac{\partial \overline{x}}{\partial P} = 0. \tag{24}$$

Concentrating on the case $s^2 = \sigma^2$ (the case of $s^2 \neq \sigma^2$ is presented in the online appendix), we can use (6) to obtain

$$\frac{\partial \overline{x}}{\partial P} = \frac{\widehat{X}(r+\lambda)}{\widehat{X} - 1}, \tag{25}$$

with $\widehat{X} = e^{\widehat{\mu\overline{x}}}$. Using this in the price first-order condition yields

$$P = \frac{\widehat{X} - 1}{\mu\widehat{X}(r+\lambda)}. \tag{26}$$

Note that the optimal price in this case of $x_0 < \overline{x}$ does not depend on the initial state x_0.

From (6) and (26), we can then determine the equilibrium P and $\overline{x}$. We can obtain the equilibrium $\overline{x}$ by the implicit equation

$$\overline{x} = \frac{\widehat{X} - 1}{\widehat{X}}\left(\frac{1}{\widehat{\mu}} + \frac{1}{\mu}\right), \tag{27}$$

which yields that the equilibrium $\overline{x}$ and equilibrium price are decreasing in λ and r and increasing in $s^2 = \sigma^2$.

Let x^* denote the solution to Equation (27). That is, (26) gives the optimal price only if $x_0 < x^*$.

If $x_0 > \bar{x}$, then the customer would buy at $t = 0$. From the analysis in the online appendix, we have that for $x > \bar{x}$,

$$\widetilde{G}(x) = \frac{\widehat{\mu}s^2/\sigma^2 - \mu(\lambda + r)/r}{\widehat{\mu}\,\frac{r(1-s^2/\sigma^2)}{\lambda} + \widetilde{\mu}\left(\frac{\lambda+r(1-s^2/\sigma^2)}{\lambda}\right) + \mu} e^{\widetilde{\mu}(\bar{x}-x)} + \frac{\lambda}{r}. \quad (28)$$

Focusing on the case of $s^2 = \sigma^2$, this simplifies to

$$\widetilde{G}(x) = \frac{1}{2}\left(\sqrt{\frac{\lambda+r}{r}} - \frac{\lambda+r}{r}\right) e^{\widetilde{\mu}(\bar{x}-x)} + \frac{\lambda}{r}. \quad (29)$$

The firm's objective is $\max_P P(1 + \widetilde{G}(x_0))$. We can obtain the first-order condition as

$$P\frac{\partial\widetilde{G}(x_0)}{\partial\bar{x}}\frac{\partial\bar{x}}{\partial P} + 1 + \widetilde{G}(x_0) = 0. \quad (30)$$

Using (6), this simplifies to the following implicit equation, which determines the optimal $\bar{x}$ as a function of x_0:

$$\frac{1}{2}\left[1 - \sqrt{\frac{\lambda+r}{r}}\right]\left[\widehat{\mu}\bar{x}\frac{\widehat{X}}{\widehat{X}-1} - 1 + \sqrt{\frac{\lambda+r}{r}}\right] e^{\widetilde{\mu}(\bar{x}-x_0)} + \frac{\lambda+r}{r} = 0. \quad (31)$$

One can then use (6) to get the optimal price. Let $h(\bar{x}, x_0) = 0$ represent (31). Then, note that $h(\bar{x}, x^*) = 0$ generates a $\bar{x} > x^*$. That is, for x_0 slightly above x^*, (31) does not specify the equilibrium $\bar{x}$. In fact, we can obtain x^{**} as the solution to $h(\bar{x}, \bar{x}) = 0$,

$$\widehat{\mu}\frac{\bar{x}\widehat{X}}{\widehat{X}-1} = \frac{\frac{\lambda}{r} + 2\sqrt{\frac{\lambda+r}{r}}}{\sqrt{\frac{\lambda+r}{r}} - 1}, \quad (32)$$

where $x^{**} > x^*$.

We can then obtain that if $x_0 < x^*$, we have $\bar{x}$ defined by (27); if $x_0 > x^{**}$, we have $\bar{x}$ defined by (31), and for $x_0 \in [x^*, x^{**}]$, we have $\bar{x} = x_0$. For an example of $s^2 = \sigma^2 = 0.25, \lambda = 0.2$, and $r = 0.05$, we can obtain $x^* \approx 2.18$, and also, $x^{**} \approx 4.84$.

We collect some of these results in the following proposition.

Proposition 4. *Consider optimal pricing and the case in which the amount of information gained while owning or not owning the product is relatively close, s^2 close to σ^2. Then, the purchase threshold is independent of x_0 for $x_0 < x^*$ and it is increasing in x_0 for $x_0 > x^*$, with $\bar{x} = x_0$ for $x_0 \in [x^*, x^{**}]$, and $\bar{x} < x_0$ for $x_0 > x^{**}$. The optimal price is decreasing in λ and r for all x_0. The purchase threshold under optimal pricing is nonincreasing in λ for all x_0, and is decreasing in r for $x_0 < x^*$, and it can be either increasing or decreasing in r for $x_0 > x^{**}$. An increase in s^2 leads to a decrease in the purchase threshold for $x_0 < x^*$, an increase (decrease) in the optimal price if λ/r is sufficiently large (small) for $x_0 < x^*$, an increase in the optimal price for $x_0 \in (x^*, x^{**})$, and an increase or decrease of the price and purchase threshold for $x_0 > x^{**}$. An increase in s^2 under the constraint of $s^2 = \sigma^2$ leads to an increase in the purchase threshold and the optimal price for $x_0 < x^*$, a decrease in the optimal price for $x_0 \in [x^*, x^{**}]$, and an increase or decrease of the price and purchase threshold for $x_0 > x^{**}$.*

When x_0 is low, the optimal price and the purchase threshold do not depend on x_0. The firm has to take into account the potential future revenue of the expected future repurchases by the consumer and therefore, does not lower the price beyond a certain level, which makes the optimal price not varying with the expected initial valuation if that valuation is sufficiently low. In that region, an increase in the duration of the product or a decrease in the discount rate makes the product more valuable, and the firm optimally increases its price, with a resulting increase in the purchase threshold. Similarly, when the amount of information gained increases, we have that both the consumer is more likely to get information that makes the consumer willing to purchase the product and the consumer has a higher potential of having greater benefits after purchase. Both of these effects then lead the firm to raise its price and the purchase threshold to increase.

Note that the effects of the discount rate and product duration on the purchase threshold are opposite from those under an exogenous price. Thus, the effects of the price increase dominate the effects of the discount rate and product duration on the consumer's purchase threshold. If price does not increase too much, then a longer product duration causes consumers to purchase earlier. Instead of keeping the price low and taking advantage of quicker purchases, the firm should increase the price enough such that consumers delay purchase

For higher x_0, the effects of the duration of the product on the purchase threshold and the optimal price are in the same direction as when x_0 is low. Similarly, if $x_0 \in [x^*, x^{**}]$, the effect of the discount rate on the optimal price is also in the same direction as in the x_0 low condition. The effect of the discount rate on the purchase threshold for $x_0 > x^{**}$ can be either positive or negative. When $x_0 > x^{**}$, the consumer starts in a region when the consumer is likely to make a repurchase when the product breaks down, which increases the effect of the discount rate on the value of the product and may make then the consumer more demanding on the purchase threshold.

The effect of the information gained when owning the product, s^2, is interesting. Recall that when price was fixed, an increase in the information gained when owning the product would make the consumer lower the purchase threshold to be able to gain further information when owning the product (Proposition 1). When the price is endogenous, we find that this effect continues to hold, as the equilibrium purchase threshold is decreasing in the information gained when owning the product. This effect under an exogenous price could give an incentive for the firm to increase its price, but we find that when $x_0 < x^*$, this only occurs if the discount rate is sufficiently low (low r) or if the likelihood of the product breaking down is sufficiently large (high λ). When the discount rate is large or the likelihood of the product breaking down is low, the consumer becomes more price sensitive, and this effect is bigger when the information gained when owning the product is larger. The consumer becomes more price sensitive when the discount rate is large because the benefits of purchase are accrued closer to the purchase. The consumer becomes more price sensitive when the likelihood of the product breaking down is smaller because the purchase decision has a longer effect. When the initial valuation is in the intermediate range, $x_0 \in (x^*, x^{**})$, the optimal price increases in the information gained when owning the product because of the benefit of the greater information gained in that case.

These comparative statics on s^2 also allow us to compare with the case in which all information is learned immediately after purchase. This then may indicate that, with gradual learning after purchase, the optimal price is lower (higher) than when the consumer learns everything immediately after purchase if $x_0 < x^*$, the discount rate is low (high), and the hazard rate of the product breaking down is high (low). If $x_0 \in (x^*, x^{**})$, the optimal price may also then be lower in the case considered than if the consumer learns everything immediately after purchase.

The effect of the information gained while owning or not owning the product (the case of the constraint $s^2 = \sigma^2$) on the optimal price when x_0 is not too low is also interesting. In the region where $x_0 \in [x^*, x^{**}]$, the purchase threshold is set at x_0. As the amount of information gained is a force for the consumer to be more demanding on the purchase threshold while keeping the price fixed (because of σ^2), the firm then has to decrease its price for the purchase threshold to remain fixed at x_0. The same effect also holds for $x_0 > x^{**}$, but in that region, the firm has some flexibility in increasing $\bar{x}$, and therefore, in that region, the purchase threshold can increase or decrease with the amount of information gained.

To illustrate the equilibrium, Figure 10 presents the evolution of the optimal price P and the resulting purchase threshold $\bar{x}$ as a function of the initial current utility x_0. Figures 11–13 illustrate how the optimal

Figure 10. (Color online) Evolution of the Optimal Price P and of the Resulting Purchase Threshold $\bar{x}$ as a Function of the Initial Current Utility x_0 for $s^2 = \sigma^2 = 0.25$, $\lambda = 0.2$, and $r = 0.05$

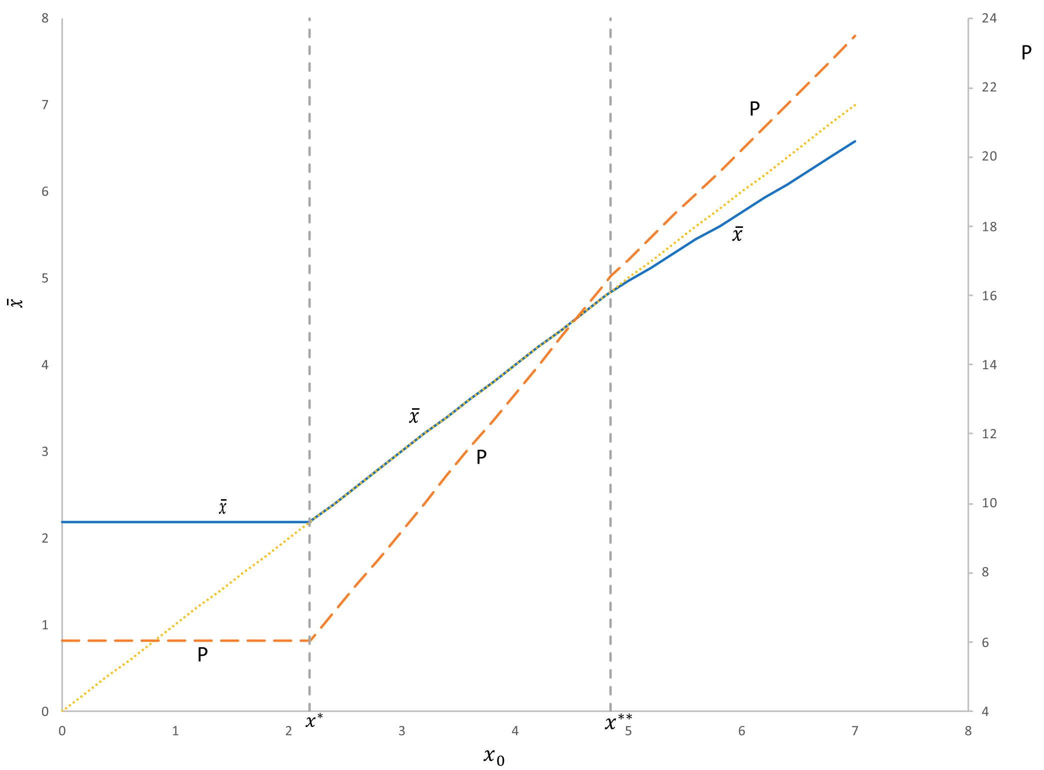
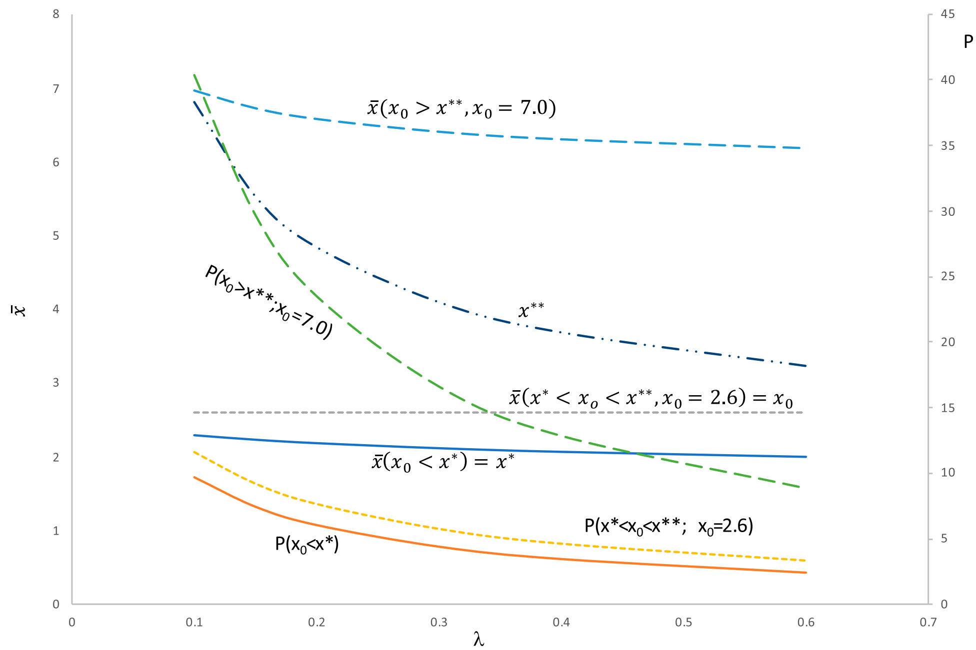


Figure 11. (Color online) Evolution of the Optimal Price P and of the Purchase Threshold $\bar{x}$ as a Function λ for $s^2 = \sigma^2 = 0.25$ and $r = 0.05$



price and the resulting purchase threshold vary with $\lambda, r,$ and $s^2 = \sigma^2$ for x_0 in the different regions, as presented in Proposition 4. Figure 12 illustrates that the purchase threshold can either increase or decrease with r for $x_0 > x^{**}$, and it presents an effect of the price declining with r for that region of x_0. Figure 13 illustrates a case where the purchase threshold increases in σ^2 with $s^2 = \sigma^2$ for $x_0 > x^{**}$, which was a possibility in the proposition, and presents that the optimal price can either increase or decrease.

Figure 12. (Color online) Evolution of the Optimal Price P and of the Purchase Threshold $\bar{x}$ as a Function r for $s^2 = \sigma^2 = 0.25$ and $\lambda = 0.2$

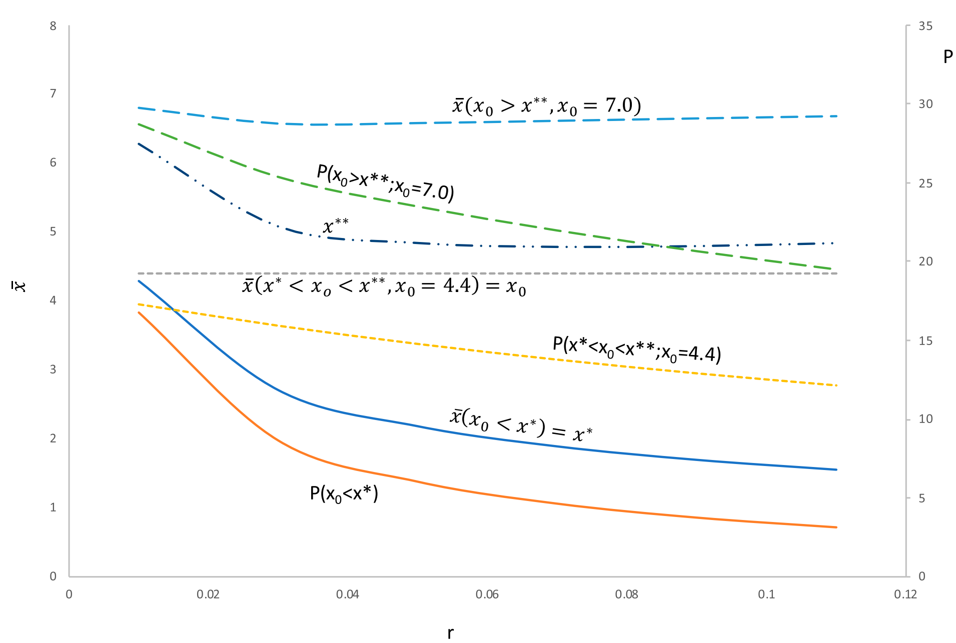
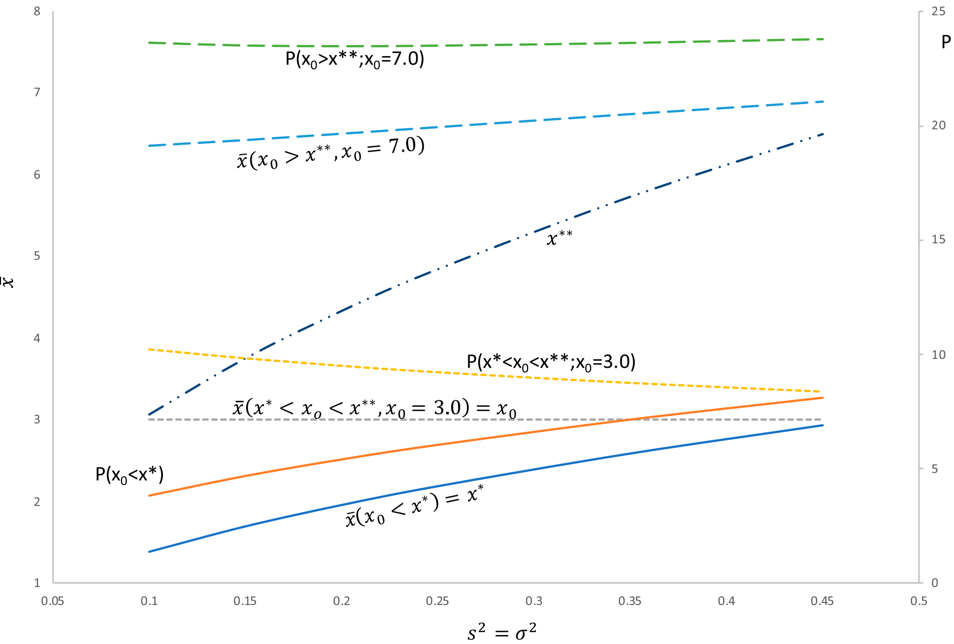


Figure 13. (Color online) Evolution of the Optimal Price P and of the Purchase Threshold $\bar{x}$ as a Function $s^2 = \sigma^2$ for $r = 0.05$ and $\lambda = 0.2$



5.2. Effect of the Product Duration on Profit

We now investigate the effect of the product duration on the firm's profit under optimal pricing. Assume that $s^2 = \sigma^2$ and x_0 is not too high so that the consumer does not buy at time 0. (The derivation of the case of $s^2 \neq \sigma^2$ is presented in the online appendix.) The optimal price is determined by (26), and the discounted number of purchases is given by (23). Then, the firm's expected profit $\Pi(x_0) = PG(x_0)$ is

$$\Pi(x_0) = \frac{1}{2}\frac{\widehat{X} - 1}{\mu\widehat{X}(r+\lambda)}e^{\mu(x_0-\bar{x})}\left[\frac{r+\lambda}{r} + \sqrt{\frac{r+\lambda}{r}}\right]. \tag{33}$$

Using (27), this becomes

$$\Pi(x_0) = \frac{\bar{x}}{2r}e^{\mu(x_0-\bar{x})}. \tag{34}$$

Taking the derivative of $\Pi(x_0)$ with respect to $\bar{x}$, we get

$$\frac{d\Pi(x_0)}{d\bar{x}} = \frac{1-\mu\bar{x}}{2r}e^{\mu(x_0-\bar{x})}. \tag{35}$$

From (27), we get that $\bar{x} \to \frac{1}{\mu}$ as $\lambda \to \infty$. Given that $\bar{x}$ decreases in λ, this implies that $\bar{x} > \frac{1}{\mu}$ for all finite λ. Thus, we have

$$\frac{d\Pi(x)}{d\lambda} = \frac{d\Pi(x)}{d\bar{x}}\frac{d\bar{x}}{d\lambda} > 0, \tag{36}$$

which means that the firm's expected profit monotonically increases in λ.

What happens at the limit of $\lambda \to \infty$? At the limit, $\bar{x} = \frac{1}{\mu}$, so the expected profit becomes

$$\Pi(x_0) = \frac{e^{\mu x_0 - 1}}{2r\mu}. \tag{37}$$

Let Pdt denote the flow price in the limit. The consumer pays for the product whenever $x - P > 0$. Thus, we have $Pdt = \bar{x}dt = \frac{1}{\mu}dt$, which means that the optimal flow price is increasing in the amount of information gained while not owning or owning the product, σ^2, and decreasing in the discount rate r.

The case of $s^2 \neq \sigma^2$, but close to each other (presented in the online appendix), has that the expected present value of profits is also increasing in λ, and we have that at the limit of $\lambda \to \infty$, $\bar{x} \to \frac{2}{\mu} - \frac{1}{\tilde{\mu}}$. The optimal flow price in that case is $Pdt = \frac{1}{\mu}dt$, and the firm's profit is

$$\Pi(x_0) = \frac{e^{\mu x_0 - 2 + \mu/\tilde{\mu}}}{r\mu(1 + \sqrt{s^2/\sigma^2})}. \tag{38}$$

Thus, at the limit, the optimal flow price is the same as when $s^2 = \sigma^2$. The optimal flow price increases in the amount of information gained while not owning the product, σ^2, decreases in the discount rate, r, and is independent at the limit of the information gained while owning the product. The purchase threshold is below the flow price if $s^2 > \sigma^2$. The consumer suffers some instantaneous utility loss to gain more information.

We summarize these results in the following proposition.

Proposition 5. *Suppose price is chosen optimally, the initial expected utility is low enough, $x_0 < 2/\mu - 1/\widetilde{\mu}$, and s^2 is close to σ^2; then, the firm's ex ante expected profit increases in λ. In the limit as $\lambda \to \infty$, the product has an infinitely short duration, the optimal price is $\sqrt{\frac{\sigma^2}{2r}}$ per unit of time, and the consumer owns the product at time t if and only if $x_t > 2/\mu - 1/\widetilde{\mu}$.*

An infinitely short duration means that the product is no longer a durable good. One can interpret Proposition 5 as suggesting that the firm prefers to sell the use of the product than the ownership of the product: that is, suggesting that the firm prefers renting over selling a product with durability. The consumer pays a flow price per period of time when he uses the product. The mechanism at work here of renting versus selling is different than that of the previous literature on the durable goods monopolist. The traditional argument for renting over selling durable goods (or decreasing durability) is that rational consumers expect that a monopolist without the ability to commit to future prices to lower price and increase supply over time to capture residual demand (e.g., Coase 1972, Bulow 1982). In Stokey (1979, 1981), a monopolist with commitment power is indifferent between renting and selling. In the current model, the firm prefers lower durability, even though it has commitment power on price, which is constant over time. If a consumer has to make an irreversible purchase decision, then there is an option value in delaying the purchase. The consumer can choose to acquire more information and preserve the option to make a purchase at a later time. Such delaying is not in the interest of the firm. With a high durability, a firm has to charge a higher price. This further exacerbates the problem because the option value of delaying the purchase is higher when the consumer has to make a more costly decision with a longer-lasting impact. Thus, higher durability results in an increase in price and in the purchase threshold and a decrease in the expected discounted lifetime value of the consumer. However, the firm can effectively eliminate this option value of delaying by switching to continuously renting the product/service. A consumer's decision to rent does not limit her future choices but only affects her current flow utility. So, there is no option value of delaying as long as the current flow utility from renting is positive.

Note that this result on the optimality of the infinitely short duration of the product holds if s^2 is close to σ^2. If s^2 is much greater than σ^2, then the optimal $\overline{x}$ may end up being negative, and then, the shortest duration possible will no longer be optimal. Note also that we assume the marginal costs of production to be zero. If the marginal cost is linear in the discounted duration the product (that is, $C(\lambda) = K/(r+\lambda)$ for some cost parameter K), then the cost can be normalized into x and P, and all results will continue to hold. However, if, for example, there is a fixed cost per transaction that is independent of λ, which can be either a production cost or a transaction cost paid by the consumer when a product breaks down and she has to buy a new one, then the infinitely short duration will no longer be optimal. Note also that if the duration decision cannot be credibly communicated to consumers or the breakdown hazard rate is correlated with the product fit, such that a product breaking down decreases the consumer's expected flow utility from using the product, then the infinitely short duration may also not be optimal.

5.3. Further Discussion

5.3.1. Product Returns. One possibility not considered is that the consumer could potentially return or resell the product at a certain price. In the case of a return, the firm could also set a return time limit. In that case, the consumer would return or resell the product if the expected valuation of the product decreased sufficiently. That is, in that case, we would need to figure out another (lower) threshold such that when the product's expected valuation reached that threshold, the consumer would return or resell the product. In the case of returns, the firm would then need to also optimize on the return price and the return time limit. Although interesting, this would bring considerable complexities into the analysis and be beyond the scope of this paper, and it can potentially be first considered in the case of an infinitely durable product.

One interesting benchmark in the case of returns could be seen as the case in which the consumer can return the product at any time (no constraints) and the return revenue to the consumer is the price originally paid. In that case, for $s^2 = \sigma^2$, the consumer just purchases if the expected current valuation reaches $(r+\lambda)P$ and keeps the product as long as the product current expected valuation stays above that threshold. In that case, we can obtain that the optimal price for a sufficiently low initial expected current valuation is $\frac{1}{\mu(r+\lambda)}$ and that the optimal product duration is infinitely small, as in Proposition 5 (the derivation is presented in the online appendix). The market forces that determine the product duration are exactly the same as when there are no returns at the optimum, and therefore, it is also optimal to have an infinitely small product duration in this case. Also, note that product return becomes irrelevant for an infinitely small product duration.

Another potential possibility for the consumer when owning the product is to dispose of it when the expected flow utility is sufficiently low. Under the assumption considered that the extent of information learned is greater when owning than when not owning the product, $s^2 > \sigma^2$, disposing is never optimal as there is more to learn while owning the product ($\widetilde{V}(x) > W(x)$, for all $x < \overline{x}$). If what is learned when not owning the product is more than what is learned when owning the product ($s^2 < \sigma^2$), it can potentially be better to dispose of the product to have a greater chance of the expected flow utility increasing sufficiently, but this case can be seen more as taking different draws of the product than regarding the central question of the paper.

5.3.2. Dynamic Pricing. Another possibility not considered here is that the firm may be able to change prices over time, which could be important to consider in a setting of longer-term behavior as considered here. Alternatively, the situation considered here with constant prices could be seen as appropriate in a setting in which consumers continuously appear in the market over time.

One potential case to analyze could be one in which we restrict the number of price changes and the timings of those price changes. Consider the firm committing to two prices: an initial price for the first purchase, P_0, and a subsequent price for all future repurchases, P_1. We denote the threshold for the first purchase with P_0 as $\overline{x}_0$ and denote the threshold for the subsequent purchases with P_1 as $\overline{x}_1$. We consider the case with $s^2 = \sigma^2$ so that $\overline{x}_0 > 0$ and $\overline{x}_1 > 0$.

There are two possibilities at time 0. If the consumer does not buy immediately at time 0, then we must have $\overline{x}_0 > x_0$. If the consumer buys immediately at time 0, then $\overline{x}_0 \le x_0$. However, if $\overline{x}_0 < x_0$, then we must still have $\overline{x}_0 < x_0$ for a marginal increase in P_0, which strictly increases the firm's profit as the consumer immediately buys under a slightly higher initial price, with the same expected profit after the initial purchase. Thus, under optimal pricing, we must have $\overline{x}_0 \ge x_0$.

Using the expected time to next purchase, $\delta(x_0)$, and the discounted number of future purchases, $\widetilde{G}(x)$, the expected profit for the firm at time 0 can be written as

$$\max_{P_0, P_1} e^{-\mu(\overline{x}_0 - x_0)}[P_0 + P_1\widetilde{G}(\overline{x}_0)]. \tag{39}$$

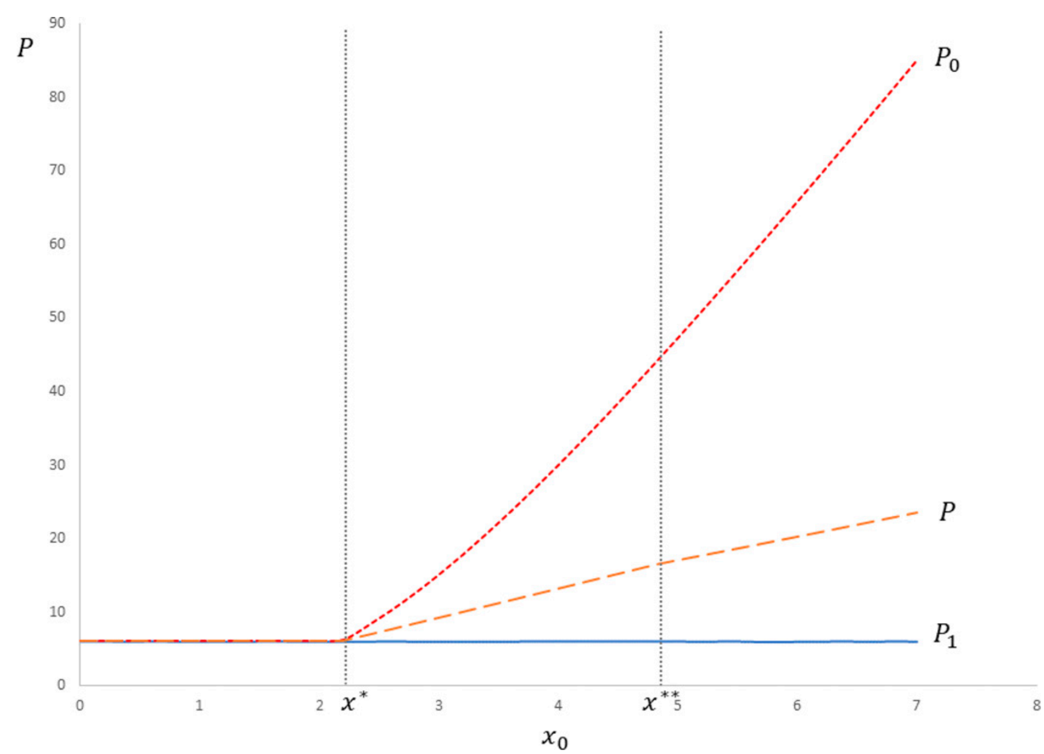
We can express P_1 as a function of $\overline{x}_1$ and express P_0 and $\widetilde{G}(\overline{x}_0)$ as functions of $\overline{x}_0$ and $\overline{x}_1$. (The derivation is available in the online appendix.) Then, the firm maximizes profit over $\overline{x}_0$ and $\overline{x}_1$:

$$\max_{\overline{x}_0 \ge x_0, \overline{x}_1} e^{-\mu(\overline{x}_0 - x_0)}[P_0 + P_1\widetilde{G}(\overline{x}_0)], \tag{40}$$

which can be solved numerically.

Figure 14 illustrates the optimal initial price, P_0, and the optimal subsequent price, P_1, for different initial positions and compares them with the optimal

Figure 14. (Color online) Optimal Dynamic Prices P_0 and P_1 and the Optimal Uniform Price P as a Function x_0 for $s^2 = \sigma^2 = 0.25$, $r = 0.05$, and $\lambda = 0.2$



uniform price, P, from Figure 10. When x_0 is low, the optimal dynamic prices are the same as the optimal uniform price, and the consumer does not purchase immediately. However, the prices diverge for $x_0 > x^*$. When the consumer has a higher initial willingness to pay, the firm extracts the additional surplus from a higher x_0 all through the initial price while keeping the subsequent price constant. As x_0 increases above x^*, the optimal subsequent price remains constant, whereas the optimal initial price increases at a faster pace than the optimal uniform price.

Note that the model assumes that the firm observes x_0 and can commit to future prices ex ante. If the firm cannot commit to future prices and is unable to observe the consumer's evolving state x_t, one has to consider the firm's belief about x_t. Consider that any $t > 0$ when the consumer does not own the product, the firm's belief about x_t follows some continuous distribution. The firm faces a skimming problem similar to that of bargaining under incomplete information (e.g., Fudenberg et al. 1985). The firm tries to learn about the consumer's valuation for the product through successive price offers. The consumer's purchase threshold also depends on the consumer's expectation of all future price offers. After each price offer, if the consumer does not buy, the firm's belief becomes truncated at the top. However, comparing with Fudenberg et al. (1985), the current model has the additional features of repeated purchases and evolving x_t, both of which significantly complicate the problem.

If the firm cannot commit to future prices and is able to observe the consumer beliefs, we are then in a situation similar to Ning (2021). The consumer may suffer from a holdup problem, in which the firm raises price as x_t increases. As in Ning (2021), we would then potentially need to allow the firm to self-impose a price ceiling in the form of a list price, with the possibility of the firm offering dynamic discounts.

6. Conclusion

This paper studies the possibility of a consumer deciding when to purchase and repurchase a product as preferences evolve over time. The paper generates rich dynamics of when a consumer owns a product and decides when to repurchase it when the product breaks down. We characterize the optimal strategy of the consumer and then, compute the market equilibrium.

The model can be seen as considering a mixture of search and experience goods, where the consumer decides when to learn information about the product prior to purchase and when learn information about the product while using it. In particular, when the consumer gains more information when using the product, the consumer becomes less demanding on the expected value of the product to decide to make a purchase but on the other hand, makes less frequent repurchases after the initial purchase.

We can construct the optimal price to charge given an initial expected current utility of the product. We find that if the initial expected current utility is low enough such that firm does not want to get the consumer to make an immediate purchase, the optimal price is independent of the initial valuation. On the other hand, when the initial expected current utility is sufficiently high, the firm may want to price such that consumers purchase the product immediately, and in that case, the optimal price is increasing in the initial expected valuation.

We find that although greater information gained increases prices for low initial valuations, it can decrease prices for higher initial valuations. We also find that if the firm could choose the product duration, it would choose the smallest one possible, which can be interpreted as a rental pricing mechanism.

It would be interesting to explore in future research the possibility of having a subscription model where consumers commit to subscribe for some period of time. It would also be interesting to study the possibility of product returns in this environment of evolving preferences, allowing for the possibility of prices evolving over time.

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Endnotes

[1] Note that it could be a service that the consumer does not receive all the benefit of visiting immediately after consumption, and there is some uncertain time when the service may again be potentially needed.

[2] See also Ke et al. (2016), Che and Mierendorff (2019), and Ke and Villas-Boas (2019) for similar learning prior to choosing one alternative when there are more than two alternatives. For empirical work on gradual learning prior to purchase, see, for example, Gardete and Antill (2020) and Ursu et al. (2020). There is literature on the effects of search behavior on product design (for example, Kuksov 2004).

[3] See also Felli and Harris (1996) for a similar setup in a labor market setting.

[4] We can also consider a simple Bayesian example. The product is worth one with probability μ_0 and worth zero with probability $1-\mu_0$. The consumer learns by receiving a binary signal of the product's worth with accuracy α and updates her belief to μ_1. Then, the expected flow utility is a martingale, as $E(\mu_1|\mu_0) = \mu_0$. If the signal is more accurate when owning the product than not owning the product, then $\text{Var}(\mu_1)$ is bigger if the consumer owns the product.

[5] We consider the case in which there is always enough to learn about the product, such that the variance of the expected flow

utility is always greater when owning the product than when not owning it if the consumer gains more information when owning the product. This captures an important dimension of the browse or experience trade-off. If what can be learned about the product is bounded with probability one, then we can have situations in which the variance of the expected flow utility after owning the product after a sufficiently long period of time is smaller than when not owning the product. As an extreme, if what is learned when owning the product is learned immediately, the variance of the expected flow utility is large at the time immediately after the purchase but is zero thereafter. The paper focuses on the case in which what is learned when owning the product is learned over time (therefore, there is always learning, or preferences are evolving).

[6] For a similar framework, see, for example, Roberts and Weitzman (1981), Moscarini and Smith (2001), Branco et al. (2012), and Fudenberg et al. (2018).

[7] The case in which the consumer learns everything at the first encounter with the product is the typical search costs model (e.g., Weitzman 1979). Note also that if learning is on attributes, it is likely that consumers choose to learn first the more important attributes, leading again to decreasing variance over time.

[8] Also as discussed, one can see the case of $s^2 = 0$ as the case of search goods as in that case after search, if the consumer purchases the product, the consumer will end up purchasing the product every time that the product breaks down (as x does not change when the consumer owns the product).

[9] This certainty equivalent time can be seen as a relevant measure of time until the next purchase in this context given that if $x < \overline{x}$, the expected length of time until the next purchase is infinity. See the online appendix for an explanation of this result.

[10] The case considered here can also be seen as the firm having information about the starting expected current utility of each consumer and doing personalized pricing. It would also be interesting to consider optimal pricing in this setup under competition, but that is beyond the scope of this paper.

[11] Note that $N(x)$ and $\widetilde{N}(x)$ represent the expected actual numbers of purchases of a consumer, whereas $G(x)$ and $\widetilde{G}(x)$ represent the expected discounted numbers of purchases of a consumer. In order to determine the present value of profits for the firm (and its optimal policy), we need to use the expected discounted number of purchases.

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